TECHNISCHE UNIVERSITÄT DRESDEN

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Playing Games: Monte Carlo Tree Search

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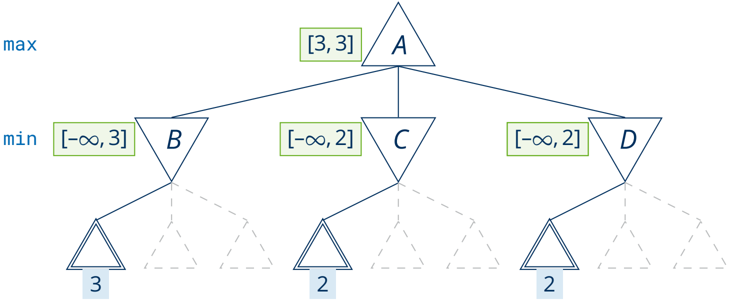
Previously ...

- Game trees can be succinctly represented by **state-based game models**.
- **Minimax Tree Search** can be used to solve sequential (two-player zero-sum) games with perfect information.
- **Alpha-Beta Pruning** allows to reduce the search space without sacrificing solutions.
- **Heuristic Evaluation** of states can be used to reduce search depth.
- Further heuristics may reduce the search space (typically with sacrifices).



Overview

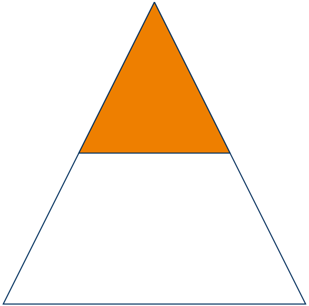
Monte Carlo Tree Search

Selection Policy: UCT

Tree Search: Shannon's Type A and Type B

In Claude Shannon's 1950 paper *Programming a Computer for Playing Chess*, he suggests two types of tree search strategies:

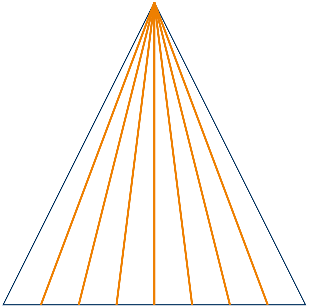
wide, but shallow



Type A

Alpha-Beta Tree Search

narrow, but deep



Type B

Monte Carlo Tree Search

Teaching Feedback

10min to fill out the feedback form:

https://tud.link/gr85

Monte Carlo Tree Search

Monte Carlo Tree Search

Terminology

A **Monte Carlo algorithm** is a randomised algorithm whose output may be incorrect with a certain (typically small) probability.

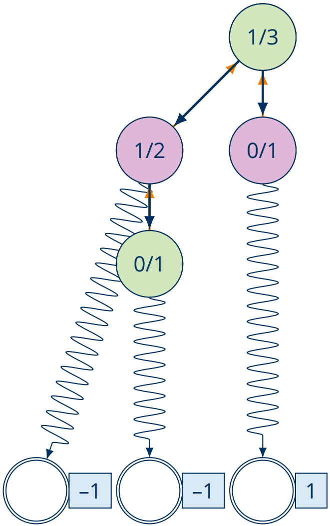
Main Idea of Monte Carlo Tree Search: Simulate random move sequences from current to terminal states and do statistics on moves leading to wins.

Some relevant notions:

- Playout: Complete move sequence from a state to a terminal state.
- Random move sequences only inform about random play, so a playout policy is needed to bias simulation towards optimal play.
- In pure Monte Carlo search, we do N simulations starting in the current state and record average payoffs for all moves.
- Selection policy: Determines from which nodes to start simulations; faces the fundamental issue to balance exploitation and exploration.

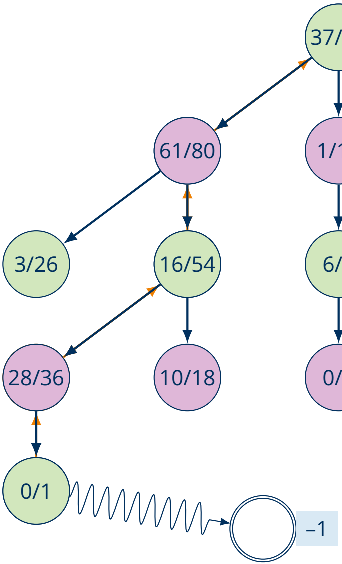
Monte Carlo Tree Search: Example (1)

1. Selection
2. Expansion
3. Simulation
4. Backpropagation



Monte Carlo Tree Search: Example (2)

1. Selection
2. Expansion
3. Simulation
4. Backpropagation



Monte Carlo Tree Search: Algorithm

function **monte-carlo-tree-search**(s : *state*) {
  tree $:=$ **get-tree-below**(s)
  while **is-time-remaining**() **do** {
    leaf $:=$ **select**(*tree*)
    child $:=$ **expand**(*leaf*)
    result $:=$ **simulate**(*child*)
    back-propagate(*tree*, *child*, *result*) }
  return **move-to-node-with-most-playouts**(*tree*) }

- **get-tree-below** returns the search tree below the node for the state
- **is-time-remaining** checks whether we are still within the time limit
- **select** uses the selection policy to find a node to expand next
- **expand** adds a new child to the given node (makes a move)
- **simulate** does a full playout, returning only the result (utility value)
- **back-propagate** propagates the result value up the search tree

Selection Policy: UCT

Selection Policy: UCT

An effective policy: UCT – "upper confidence bounds applied to trees".

UCT ranks moves according to their "upper confidence bound" value.

Definition

The **upper confidence bound** value for a node n is obtained thus:

$$\mathrm{UCB1}(n) := \frac{U(n)}{N(n)} + c \cdot \sqrt{\frac{\ln N(n')}{N(n)}}$$

where

- n' is the unique parent of n in the search tree,
- $U(n)$ is the total utility of node n (summed up over all playouts),
- $N(m)$ is the total number of playouts through node m,
- c is a constant that is typically chosen empirically (theoretically $c = \sqrt{2}$).

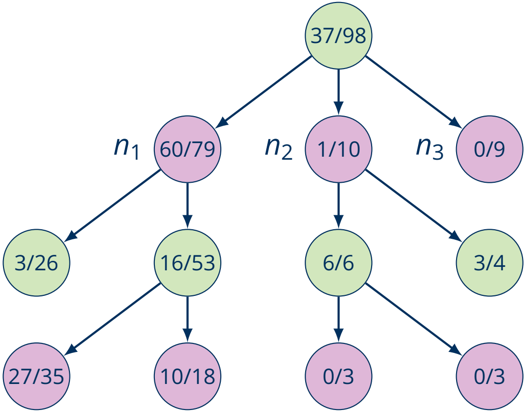
Constant c balances exploitation (first fraction) and exploration (square root).

UCT: Example

$$\mathrm{UCB1}(n_1) = \frac{60}{79} + \sqrt{\frac{2 \cdot \ln 98}{79}}$$
$$\approx 0.76 + 0.34 = 1.1$$

$$\mathrm{UCB1}(n_2) = \frac{1}{10} + \sqrt{\frac{2 \cdot \ln 98}{10}}$$
$$\approx 0.1 + 0.96 = 1.06$$

$$\mathrm{UCB1}(n_3) = \frac{0}{9} + \sqrt{\frac{2 \cdot \ln 98}{9}}$$
$$\approx 1$$



Digression: Multi-Armed Bandits (1)

- A K**-armed bandit problem** is given by random variables $X_{i,n}$ for $1 \leq i \leq K$ and $n \geq 1$, where each i is the index of a gambling machine (the "arm" of a bandit).
- Successive plays of machine i yield rewards $X_{i,1}, X_{i,2}, \ldots$ which are independent and identically distributed according to an unknown law with unknown expectation μ_i.
- Rewards across machines are also independent (and not identically distributed): $X_{i,s}$ and $X_{j,t}$ are independent for $1 \leq i < j \leq K$ and $s, t \geq 1$.
- A **policy** is a function mapping past plays and rewards to the next arm to play.
- The **regret** of a policy is the difference between the maximally possible payoff and the actually obtained payoff.

Digression: Multi-Armed Bandits (2)

- UCB1 is a specific policy of "playing" multi-armed bandits that achieves logarithmic regret (in the number n of plays; known to be optimal):

deterministic policy ucb1:

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initialisation: play each machine once

loop:

play machine j that maximises $\bar{x}_j + \sqrt{\frac{2 \ln n}{n_j}}$ where

- $\bar{x}_j$ is the average reward obtained from machine j,
- n_j is the number of times machine j has been played so far,
- n is the overall number of plays done so far.

MCTS with UCT: Remarks

- The definition of the UCB1 values guarantees that the node with the highest number of playouts is also the one with the highest average utility.
- In addition, the number of playouts also reflects the confidence in the average utility value.
- The time to compute the result of a playout is linear in the height of the game tree.
- We still need a playout policy to achieve “realistic” playout values.
- AlphaZero [Silver et al., 2018] learns a playout policy from self-play using neural networks (interleaving learning and MCTS).

Advances in Computer Go

- Go is estimated to have 10^{172} states and a branching factor of at least 361
- Heuristic evaluation of states is not very effective because material value is not very important and most positions are in flux until the endgame

$\rightsquigarrow$ Alpha-Beta Tree Search is not well-suited for Go playing

- Go-playing AIs were weak (beaten by humans) until the late 2000s
- Monte Carlo Tree Search [Coulom, 2007] improved computer Go playing
- Adaptive Multistage Sampling (AMS) algorithm incorporated UCB1 into Monte Carlo sampling [Chang et al., 2005]
- UCT algorithm incorporated UCB1 into MCTS [Kocsis & Szepesvári, 2006]
- Deep reinforcement learning to obtain a playout policy [Silver et al., 2018]
- Computer victory (AlphaGo) over human champions (2015 Fan Hui, 2016 Lee Sedol, 2017 Ke Jie)

The End?

Example: Attackability

Gleave et al. [2023] recently presented an attack on a super-human Go AI:

- Using reinforcement learning against a fixed victim (KataGo), they are able to discover systematic weaknesses in KataGo's gameplay.
- They use AlphaZero-style training, but where AZ plays against itself, they train an attacker to play against KataGo.
- The trained attacker achieves significant win rates against the victim, with and without search.
- The discovered exploit is interpretable and can be learnt by (expert) human players, who can then in turn reliably win against KataGo.

$\rightsquigarrow$ If there are single moves that can turn the game, MCTS might fail to consider those moves due to its stochastic mode of operation.

Conclusion

Summary

- **Monte Carlo Tree Search** uses random playouts to evaluate moves and keeps statistics on which moves led to which payoffs how many times.
- A **selection policy** balances exploitation and exploration.
- **UCT** is an effective selection policy that applies UCB1 to trees.
- A **playout policy** steers playout simulations towards realistic play.
- MCTS and deep reinforcement learning led to expert-level Go programs.

Action Points

- Implement a MCTS-based program for playing Tic-Tac-Toe.