

Exercise Sheet 4: Conjunctive Queries, CSP, and Hypergraphs

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Exercise 4.1. Decide if the following conjunctive queries are tree queries by applying (one version of) the GYO algorithm.

1. $\exists x, y, z, v. r(x, y) \wedge r(y, z) \wedge r(z, v) \wedge s(x, y, z) \wedge s(y, z, v)$
2. $\exists x, y, z, u, v, w. r(x, y) \wedge s(x, z, v) \wedge r(u, z) \wedge t(x, v, u, w)$

Exercise 4.2. In the lecture it was mentioned that adding equality and unions of conjunctive queries increases the expressive power of conjunctive queries. Why is that the case?

Hint:

Give an argument similar to the ones from exercise 1.6 (first exercise sheet).

Exercise 4.3. Solve the following combinatorial crossword puzzle using Yannakakis' algorithm (in spirit). Specify the join tree that you are using.

x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_8		x_9				x_{10}
x_{11}		x_{12}		x_{13}	x_{14}	x_{15}
x_{16}		x_{17}				x_{18}
x_{19}		x_{20}		x_{21}	x_{22}	x_{23}

1 hor.:

B	R	I	S	T	O	L
C	A	R	A	M	E	L
P	H	A	R	A	O	H
S	P	I	N	A	C	H
T	S	U	N	A	M	I

1 vert.:

C	L	E	A	R
H	U	M	A	N
P	E	A	C	E
S	H	A	R	K
T	I	G	E	R

3 vert.:

H	A	P	P	Y
I	N	F	E	R
L	A	B	O	R
L	A	T	E	R
U	N	T	I	L

7 vert.:

H	E	A	R	T
H	O	N	E	Y
I	R	O	N	Y
L	O	G	I	C
M	A	G	I	C

13 hor.:

A	N	D
C	A	T
D	I	M
L	A	G
W	I	N

21 hor.:

A	R	C
F	E	E
L	O	W
T	W	O
W	A	Y

Exercise 4.4. It was shown in the lecture that the 3-colourability problem for graphs can be reduced to the homomorphism problem. Therefore, it can also be expressed as a BCQ answering problem. In which cases is the resulting BCQ a tree query? What is the complexity of solving the 3-colourability problem for these cases?