Deduction Systems

Tutorial 1

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Exercise 1.1. Recap the notions "theory", "logical consequence" and "equivalence" and decide if the following claims are true or false for FOL. Give an informal justification for your answer.
For arbitrary theories $\mathcal{T}$ and $\mathcal{S}$ holds:

(a) If a formula (axiom) F is generally valid, then $\mathcal{T} \models F$, i.e., every theory has at least all tautologies as consequence.

(b) The more axioms a theory contains the more models it has. More precisely: if $\mathcal{T} \subseteq \mathcal{S}$, then every model of $\mathcal{T}$ is a model of $\mathcal{S}$.

(c) The more axioms a theory contains, the more logical consequences it has. More precisely, if $\mathcal{T} \subseteq \mathcal{S}$, then every logical consequence from $\mathcal{T}$ is also a consequence from $\mathcal{S}$.

(d) If $\neg F \in \mathcal{T}$, then $\mathcal{T} \models F$ can never hold(F being an arbitrary formula).

(e) If two theories differ syntactically ($\mathcal{T} \neq \mathcal{S}$), then they differ in at least one logical consequence (e.g., through the existence of a formula F with $\mathcal{T} \models F$ but $\mathcal{S} \not\models F$).

Exercise 1.2. Model the following statements as description logic axioms:

- Any `Vegetable` is a `PizzaTopping`.
- Nothing can be a `PizzaTopping` and a `Pizza` at the same time.
- The individual `aubergine` is an instance of the concept `Vegetable`.
- Pizzas always have at least two toppings.
- Every Pizza of type `PizzaMargarita` has `Tomato` as topping.
- No Pizza of type `PizzaMargarita` has a topping of type `Meat`.

Exercise 1.3. We want to define the concept $\texttt{VegetarianPizza}$. Which of the following definitions are appropriate for this? Provide a natural language description for each of the logical statements.

(a) $\texttt{VegetarianPizza} \equiv \texttt{Pizza} \sqcap \neg\exists\texttt{hasIngredient}.(\texttt{Meat} \sqcap \texttt{Fish})$

(b) $\texttt{VegetarianPizza} \equiv \texttt{Pizza} \sqcap \forall\texttt{hasTopping}.(\neg\texttt{Meat} \sqcup \neg\texttt{Fish})$

(c) $\texttt{VegetarianPizza} \equiv \texttt{Pizza} \sqcap \neg\exists\texttt{hasTopping}.\texttt{Meat} \sqcap \neg\exists\texttt{hasTopping}.\texttt{Fish}$

(d) $\texttt{VegetarianPizza} \equiv \texttt{Pizza} \sqcap \exists\texttt{hasTopping}.\neg\texttt{Meat} \sqcap \exists\texttt{hasTopping}.\neg\texttt{Fish}$

(e) $\texttt{VegetarianPizza} \equiv \texttt{Pizza} \sqcap \forall\texttt{hasIngredient}.(\neg\texttt{Meat} \sqcap \neg\texttt{Fish})$

Exercise 1.4. Let the following knowledge base be given:

$$\texttt{hasTopping} \sqsubseteq \texttt{hasIngredient} \qquad \exists\texttt{hasTopping}.\top \sqsubseteq \texttt{Pizza}$$
$$\texttt{Vegetable} \sqcap \texttt{Cheese} \sqsubseteq \bot \qquad \texttt{Cheese} \sqcap \texttt{Meat} \sqsubseteq \bot$$
$$\texttt{Vegetable} \sqcap \texttt{Meat} \sqsubseteq \bot \qquad \texttt{Cheese} \sqcap \texttt{Fish} \sqsubseteq \bot$$
$$\texttt{Vegetable} \sqcap \texttt{Fish} \sqsubseteq \bot \qquad \texttt{Meat} \sqcap \texttt{Fish} \sqsubseteq \bot$$
$$\top \sqsubseteq \forall\texttt{hasTopping}.\texttt{PizzaTopping}$$

Consider the following additional class definitions:

$$\begin{aligned}
\texttt{CheesePizza} &\equiv \texttt{Pizza} \sqcap \exists\texttt{hasTopping}.\texttt{Cheese}\\
\texttt{PizzaSpinach} &\equiv \exists\texttt{hasTopping}.\texttt{Spinach} \sqcap \exists\texttt{hasTopping}.\texttt{Cheese} \sqcap\\
&\qquad \forall\texttt{hasTopping}.(\texttt{Spinach} \sqcup \texttt{Cheese})\\
\texttt{PizzaCarnivorus} &\equiv \texttt{Pizza} \sqcap \forall\texttt{hasTopping}.(\texttt{Meat} \sqcap \texttt{Fish})\\
\texttt{EmptyPizza} &\equiv \texttt{Pizza} \sqcap \neg\exists\texttt{hasTopping}.\top
\end{aligned}$$

(a) Which of the concepts given above would be subsumed by $\texttt{VegetarianPizza}$ by a DL reasoner (according to a correct definition from Exercise 1.3? Explain your answers.

(b) The concept definition from (a) shows that some of the pizza definitions do not model the intended concept. How could the definition be corrected?

(c) How would the result from (a) change, if one would use just $\sqsubseteq$ instead of $\equiv$ when defining $\texttt{VegetarianPizza}$?

Exercise 1.5. Assume a vocabulary with the individual names $\texttt{bonny}$ and $\texttt{clyde}$, the concept names $\texttt{Honest}$, $\texttt{Wise}$, $\texttt{Crime}$ and $\texttt{Human}$ as well as the role names $\texttt{commits}$, $\texttt{marriedWith}$, $\texttt{suspects}$, $\texttt{report}$ and $\texttt{know}$. Model the following sentences as DL axioms:

(a) Everybody, who is honest and who commits a crime, reports himself.

(b) Who is wise and honest, doesn't commit crimes.

(c) Bonnie does not report Clyde.

(d) Clyde has committed at least 10 crimes.

(e) Bonnie and Clyde have committed at least one crime together.

(f) Everybody knowing a suspect, is a suspect himself.

Exercise 1.6. Transform the following concepts into negation normal form:

(a) $\neg(A \sqcap \forall r.B)$

(b) $\neg\forall r.\exists s.(\neg B \sqcup \exists r.A)$

(c) $\neg((\neg A \sqcap \exists r.\top) \sqcup \geqslant 3\, s.(A \sqcup \neg B))$