4. Bisimilarity for Processes with Internal Activities

Lecture on Models of Concurrent Systems

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Stephan Mennicke May 3-10, 2022

Weak Transition Relation

Let us denote by Act an action alphabet, excluding the internal action τ .

By Act_{τ} we denote the set $Act \cup \{\tau\}$.

Definition 4.1: We call an LTS $(Pr, Act_{\tau}, \rightarrow)$ a weak LTS as it may use internal actions between processes. Likewise, an LTS (Pr, Act, \rightarrow) is a strong LTS.

We may turn every weak LTS into a strong LTS by abstracting from internal transitions. Thereby, so-called weak transition relations are used.

Definition 4.2: For LTS $(Pr, Act_{\tau}, \rightarrow)$ define its strong version by

$$(Pr, Act, \Rightarrow \cup \bigcup_{\alpha \in Act} \stackrel{\alpha}{\Longrightarrow}),$$

where $\Rightarrow := \stackrel{\tau}{\rightarrow}^*$ and $\stackrel{\alpha}{\Rightarrow} := \Rightarrow \stackrel{\alpha}{\rightarrow} \Rightarrow$.

Stephan Mennicke

Concurrency Theory

Weak Bisimilarity

Definition 4.3: A process relation \mathcal{W} is a **weak bisimulation** if, for all $(P,Q) \in \mathcal{W}$ and $\alpha \in Act$,

- 1. for all P' with $P \xrightarrow{\alpha} P'$, there is a Q', such that $Q \xrightarrow{\alpha} Q'$ and $(P', Q') \in \mathcal{W}$;
- 2. for all P' with $P \Rightarrow P'$, there is a Q', such that $Q \Rightarrow Q'$ and $(P', Q') \in \mathcal{W}$;
- 3. for all Q' with $Q \xrightarrow{\alpha} Q'$, there is a P', such that $P \xrightarrow{\alpha} P'$ and $(P', Q') \in \mathcal{W}$;
- 4. for all Q' with $Q \Rightarrow Q'$, there is a P', such that $P \Rightarrow P'$ and $(P', Q') \in \mathcal{W}$.

Processes P and Q are **weakly bisimilar**, denoted $P \iff Q$, if, and only if, there is a weak bisimulation \mathcal{W} , such that $(P, Q) \in \mathcal{W}$.

Theorem 4.4: Weak bisimilarity is an equivalence relation.

However, weak bisimilarity is not a congruence for CCS. More specifically, weak bisimilarity may fail in choice contexts: Consider weakly bisimilar processes P = a.0 and $Q = \tau.a.0$ (proof: $\mathcal{W} = \{(a.0, \tau.a.0), (a.0, a.0), (0, 0)\}$ is a weak bisimulation). However, $C[P] \not \leftrightarrow C[Q]$ for $C = \bullet + b.0$.

Weak bisimilarity does not recognize the first change of states of C[Q], disabling b.

Observational Congruence (aka. Rooted Weak Bisimilarity)

Fix: Make the very first move observable! Here, we allow for $\stackrel{\tau}{\Rightarrow}$ -transitions, being $\Rightarrow \stackrel{\tau}{\rightarrow} \Rightarrow$.

Definition 4.5: Observational congruence is the largest relation, such that $P \iff {}^cQ$ if, and only if, for all $\alpha \in Act_{\tau}$

1. $P \xrightarrow{\alpha} P'$ implies there is a Q', such that $Q \xrightarrow{\alpha} Q'$ and $P' \iff Q'$, and

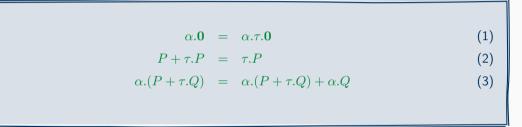
2. $Q \xrightarrow{\alpha} Q'$ implies there is a P', such that $P \xrightarrow{\alpha} P'$ and $P' \iff Q'$.

Now, P = a.0 and $Q = \tau.a.0$ are not observationally congruent anymore, as $Q \xrightarrow{\tau} a.0$, but P has only a enabled.

Theorem 4.6: Observational congruence is a congruence for CCS.

τ -Laws

The following equalities hold for $= \in \{ \nleftrightarrow, \bigstar^c \}$:



Note, for weak bisimilarity, law (2) can be adapted to $P + \tau P = P$, while this is impossible for observational congruence.

One Last Thing: Expansion Lemma

Definition 4.7: Let I be a finite index set. A process of the form $\sum_{i \in I} \alpha_i P_i$ is in head standard form.

The following result is known as the **Expansion Lemma**:

Theorem 4.8: For processes $P = \sum_{i \in I} \alpha_i P_i$ and $Q = \sum_{j \in J} \beta_j Q_j$, $P \parallel Q \cong \sum_{i \in I} \alpha_i P_i \parallel Q' + \sum_{j \in J} \beta_j P_i P \parallel Q_j + \sum_{\alpha_i = \overline{\beta_j}} \tau P_i \parallel Q_j$.

Consequence: Parallel composition can be implemented by the choice operator.