Foundations of Knowledge Representation
Horn Logics and Datalog Problems

Problem 1. *Consider the following set of formulae in propositional logic:*

1. $A \vee \neg A$
2. $(\neg A \vee B) \wedge A \wedge (\neg C \vee A \vee B) \wedge \neg B$
3. $(A \vee B) \wedge (A \vee \neg B \vee \neg C) \wedge (\neg A \vee B \vee \neg C)$

Decide which of these formulae are Horn-formulae.

Problem 2. *Consider the following knowledge bases (1) in propositional logic and (2) in first-order logic:*

Knowledge Base (1):

- $Cinema \vee SwimmingPool$
- $Cinema \rightarrow Fun$
- $SwimmingPool \vee Sleeping\_In \rightarrow Fun$

Knowledge Base (2):

- $\forall x GradStudent(x) \rightarrow Student(x))$
- $\forall x Student(x) \rightarrow HardWorker(x)$
- $GradStudent(Lisa)$

Use Resolution to:

1. *Show* $\mathsf{KB\_1} \models Fun$.
2. *Decide whether* $\mathsf{KB\_2} \models HardWorker(Lisa)$ *holds.*

Problem 3. *Assume that* $\mathsf{Parent}(x, y)$ *is true iff (if and only if)* x *is a parent of* y, $\mathsf{Male}(x)$ *is true iff* x *is male and* $\mathsf{Female}(x)$ *is true iff* x *is female. Write down Datalog rules that specify membership conditions for the binary familial relationship predicates* Father, Mother, Son, Daughter, Sibling, Brother, Sister, Grandparent, *and* Uncle. *For example:*

$$\mathsf{Parent}(x, y) \wedge \mathsf{Male}(x) \rightarrow \mathsf{Father}(x, y)$$

Make the specification as precise as possible; e.g., in the case of Father, *also add:*

$$\mathsf{Father}(x, y) \rightarrow \mathsf{Parent}(x, y)$$
$$\mathsf{Father}(x, y) \rightarrow \mathsf{Male}(x)$$

Try to be more succinct by reusing the specified predicates; e.g., use Sibling *in the specification of* Brother *and* Sister. *Explain the nature of the problem in any case where expressivity limitations of Datalog make exact specification impossible.*

Problem 4. *Consider the Datalog knowledge base* $\mathcal{K} = \langle \mathcal{R}, \mathcal{F} \rangle$ *where* $\mathcal{F}$ *contains the following facts:*

$$\mathcal{F} = \{\mathsf{Father}(john, mary), \mathsf{Mother}(mary, peter), \mathsf{Father}(john, david)\}$$

and $\mathcal{R}$ *contains the following rules:*

$$\mathsf{Parent}(x, y) \wedge \mathsf{Parent}(y, z) \rightarrow \mathsf{GrandParent}(x, z) \quad (1)$$
$$\mathsf{Parent}(x, y) \wedge \mathsf{Parent}(x, z) \rightarrow \mathsf{Sibling}(y, z) \quad (2)$$
$$\mathsf{Father}(x, y) \rightarrow \mathsf{Parent}(x, y) \quad (3)$$
$$\mathsf{Mother}(x, y) \rightarrow \mathsf{Parent}(x, y) \quad (4)$$

Do the following:

1. *Using backward-chaining show that John is a grand parent of Peter.*
2. *Using forward-chaining show that Mary and David are siblings.*