



PROBLEM SOLVING AND SEARCH IN ARTIFICIAL INTELLIGENCE

Lecture 9 Evolutionary Algorithms

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Agenda

- 1 Introduction
- 2 Uninformed Search versus Informed Search (Best First Search, A* Search, Heuristics)
- 3 Local Search, Stochastic Hill Climbing, Simulated Annealing
- 4 Tabu Search
- 5 Answer-set Programming (ASP)
- 6 Constraint Satisfaction Problems (CSP)
- 7 **Evolutionary Algorithms/ Genetic Algorithms**
- 8 Structural Decomposition Techniques (Tree/Hypertree Decompositions)

Outline

- 1 Motivation
- 2 Structure of EAs
- 3 Components of EAs
- 4 Working of EAs
- 5 Conclusion

Motivation

- Search algorithms so far **modified** (resp. constructed) **one single solution**.
- Process a **complete solution** or **construct the final solution** from smaller building blocks.
- There is a **single best solution** to be improved.

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New Idea

- Work on a **population** of solutions
- Let the solutions **compete** against each other
- Use **random variation** to search for new solutions



Rabbits and Foxes



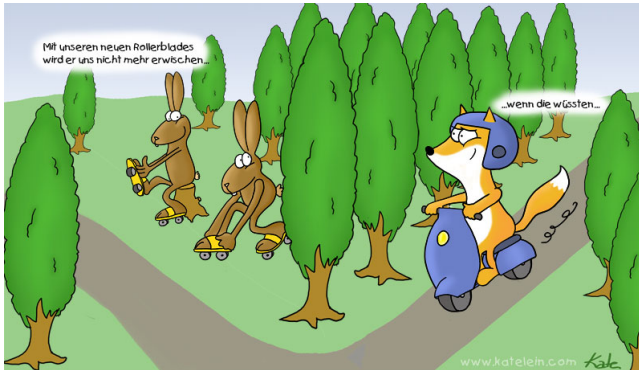
- Some rabbits are faster and smarter - they don't get eaten by foxes
- They do what rabbits do best: **make more rabbits**
- Breeding mixes the rabbits' genetic material
- Every once in a while: **mutation**
- Over generations, rabbits become faster and smarter

Rabbits and Foxes ctd.



- The same happens with foxes
- They are forced to get better at finding a meal

Rabbits and Foxes ctd.



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Evolutionary Algorithms (EAs)

- A population of individuals exists in an environment with **limited resources**
- **Competition** for resources causes selection of **fitter** individuals that are better adapted to environment
- These individuals act as seeds for generation of new individuals through **recombination** and **mutation**
- New individuals have their fitness evaluated and compete (possibly also with parents) for survival
- Over time **natural selection** causes a rise in the fitness of the population

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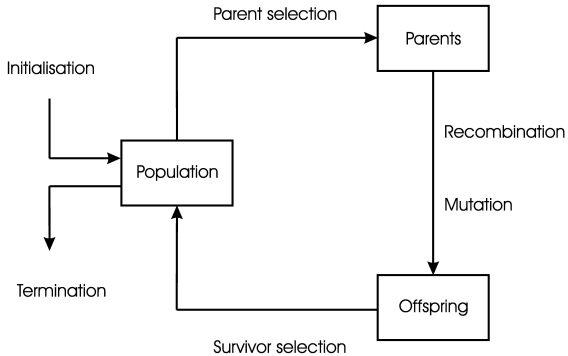
Facts on EAs

- EAs are **generate and test** algorithms
- They are **stochastic** and **population-based**
- **Variation operators** (recombination and mutation) create necessary **diversity**
- **Selection** reduces diversity and acts as a force pushing quality

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- 2 **Structure of EAs**
 - **General Schema**
 - Pseudo-Code
- 3 Components of EAs
 - Representation
 - Evaluation Function
 - Population
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General Schema of EAs



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Structure of an EA

Algorithm 1: evolutionary algorithm

```
INITIALISE population with random candidate solutions
EVALUATE each candidate
while not TERMINATION-CONDITION is satisfied do
    SELECT parents
    RECOMBINE pairs of parents
    MUTATE the resulting offspring
    EVALUATE new candidates
    SELECT individuals for the next generation
end while
```

Types of EAs

Historically different types of EAs have been associated with different representations.

- Binary strings: Genetic Algorithms
- Real-valued vectors: Evolution Strategies
- Finite state machines: Evolutionary Programming
- Trees: Genetic Programming

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Technically

- choose representation to suit problem
- choose variation operators to suit representation
- selection operators only use fitness – so they are independent of representation

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Components of EAs

- Representation (definition of individuals)
- Evaluation function (or fitness function)
- Population
- Parent selection mechanism
- Variation operators, recombination and mutation
- Survivor selection mechanism (replacement)

Necessary

- Initialisation procedure
- Termination condition

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Representation

- Candidate solutions (individuals) exist in **phenotype space**
- They are encoded in **chromosomes**, which exist in genotype space
 - Encoding: phenotype \rightarrow genotype (not necessarily one-to-one)
 - Decoding: genotype \rightarrow phenotype (must be one-to-one)
- Chromosomes contain **genes**, which are in (usually fixed) positions called **loci** (sing. locus) and have a value (**allele**)

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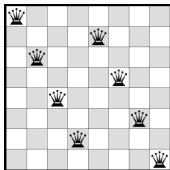
To find global optimum, every feasible solution must be represented in genotype space.

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Example (8-Queens)



1	3	5	7	2	4	6	8
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- **Phenotype**: a board configuration
- **Genotype**: a **permutation** of the numbers 1 – 8

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Evaluation (Fitness) Function

- Represents the requirements the population should adopt to
- aka quality function or objective function
- Assigns a single real-valued fitness to each phenotype which forms the basis for selection
- Typically we talk about fitness being maximized

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Example (8-Queens ctd.)

- Penalty of one queen: number of queens she can check
- Penalty of a configuration: sum of penalties of all queens
- ⇒ Penalty needs to be minimized
- ⇒ Fitness of a configuration: inverse penalty to be maximized

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Population

- Holds (representations of) possible solutions
- Usually has a fixed size and is a multiset of genotypes
- Some sophisticated EAs also assert a spatial structure on the population e.g. a grid
- Selection operators usually take whole population into account i.e. reproductive probabilities are **relative** to **current** generation
- **Diversity** of a population refers to the number of different solutions
- No single measure for diversity exists
- Typically one refers to number of different fitness values/phenotypes/genotypes present



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Parent Selection Mechanism



- Distinguish among individuals based on their quality – allow better individuals to become parents of next generation
- Individual is a parent if it has been selected to create offspring
- Responsible for pushing quality improvements
- Usually probabilistic
 - high quality solutions more likely to become parents than low quality
 - BUT: not guaranteed
 - even worst in current population has non-zero probability of becoming a parent
- Stochastic nature can aid escape from local optima

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Example (8-Queens ctd.)

Pick 5 parents randomly and take the two best to generate offspring.

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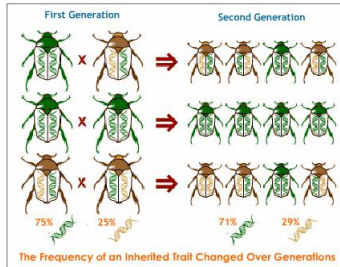
Variation Operators

- Role is to generate new candidate solutions
- Usually divided into two types according to their arity

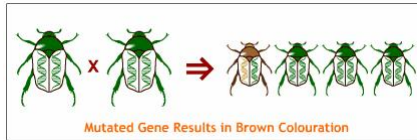
Arity 1: **mutation** operator

Arity >1: **recombination** operator; arity=2 typically called **crossover**

- Debate about relative importance of recombination and mutation
 - Nowadays most EAs use both
 - Choice of particular variation operator is representation dependent

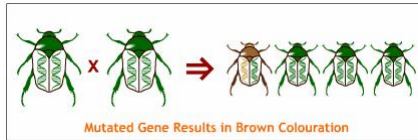


Mutation



- Acts on one genotype and delivers another
 - Element of randomness is essential and differentiates it from other unary heuristic operators
 - Generating a child amounts to stepping to a new point in search space
- ⇒ Mutation may guarantee connectedness of search space

Mutation



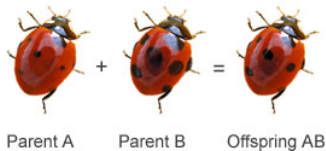
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Example (8-Queens ctd.)

Swap values of two randomly chosen positions

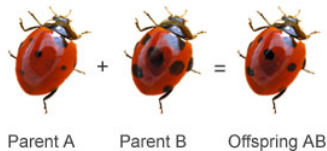


Recombination



- Merges information from parents into offspring
- Choice of what information to merge is stochastic
- Most offspring may be worse, or the same as parents
- Hope that some are better by combining elements of genotypes that lead to good traits

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Note

- Variation operators are representation dependent
- Different representations require different variation operators

Example (8-Queens ctd.)

Combine two permutations into two new permutations

- Choose random crossover point
- Copy first parts into children
- Create second part by inserting values from other parent
 - in order they appear there
 - beginning after crossover point
 - skipping values already in child

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8	7	6	5	4	3	2	1

 \Rightarrow

1	3	5	4	2	8	7	6
8	7	6	2	4	1	3	5

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Children inherit genetic material from both parents!

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Survivor Selection

- aka **replacement**
- Most EAs use fixed population size
- Often deterministic
 - Fitness-based: e.g. **rank** parents and offspring and take the best
 - Age-based: make as many offspring as parents and delete all parents
 - Combinations of the former two (elitism)

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Example (8-Queens ctd.)

Merge population and offspring – rank them according to fitness – delete the worst two.

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Initialization/Termination

Initialization

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- Needs to ensure even spread and mixture of possible allele values
- Can include existing solutions, or use problem-specific heuristics to **seed** the population

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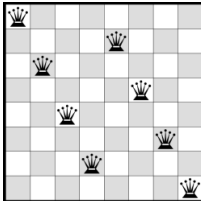
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Termination Condition

- Checked every iteration
- Reaching some (known/hoped for) fitness
- Reaching some maximum allowed number of generations
- Reaching some minimum level of diversity
- Reaching some specified number of generations without fitness improvement

Example (8-Queens ctd.)



- **Initial population:** randomly generated permutations
- **Termination condition:** solution or 10000 fitness evaluations
- **Population size:** 100
- **Recombination probability:** 100%
- **Mutation probability:** 80%

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Working of EAs

Typical **progress** of an EA illustrated in terms of **population distribution**

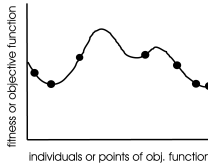


Figure : begin

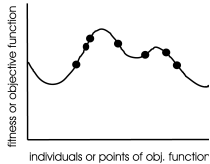


Figure : halfway

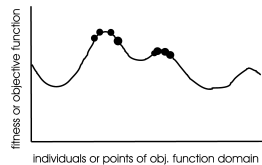


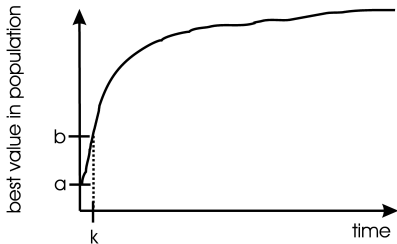
Figure : end

Working of EAs ctd.

Typical progress in terms of development of **best fitness value** within population in time.

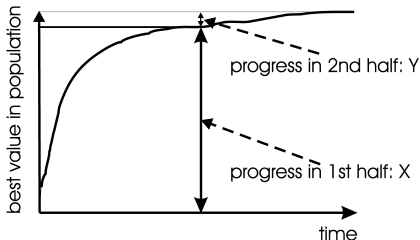


No Need for Heuristic Initialisation



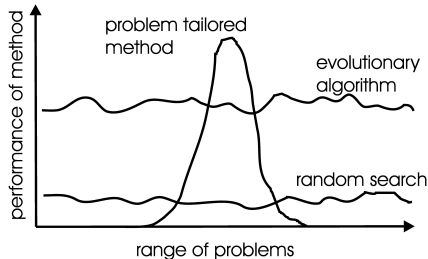
- Level a : best fitness in a randomly initialised population
- Level b : heuristic initialisation
- After k generations same level reached

Termination Conditions



- Divide the run into two equally long sections
- Fitness increases in the first half X
- Progress in second half Y is much smaller
- Due to **anytime behaviour**, efforts spent after a certain time may not result in better solution quality

Performance from Global Perspective



- Performance on a wide range of problems
- EAs are robust problem solving tools
- For most problems a problem-specific tool may
 - perform better than a generic search algorithm on most instances
 - have limited utility
 - not do well on all instances
- EAs provide an evenly good performance over a range of problems and instances

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Summary

- Idea for EAs come from evolution theory
- Components
 - Representation (definition of individuals)
 - Evaluation function (or fitness function)
 - Population
 - Parent selection mechanism
 - Variation operators, recombination and mutation
 - Survivor selection mechanism (replacement)
 - Initialisation and termination condition
- Performance

References



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