## **Deduction Systems**

## Tutorial 2

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**Exercise 2.1.** Transform the following concepts into negation normal form:

- (a)  $\neg (A \sqcap \forall r.B)$
- (b)  $\neg \forall r. \exists s. (\neg B \sqcup \exists r. A)$
- (c)  $\neg ((\neg A \sqcap \exists r. \top) \sqcup \geqslant 3 s.(A \sqcup \neg B))$

**Exercise 2.2.** Apply the tableau algorithm in order to check if the axiom  $A \sqsubseteq B$  is a logical consequence of the TBox  $\{\neg C \sqsubseteq B, A \sqcap C \sqsubseteq \bot\}$ .

**Exercise 2.3.** Apply the tableau algorithm in order to check satisfiability of the concept  $A \sqcap \forall r.B$  w.r.t. the TBox  $\{A \sqsubseteq \exists r.A, B \sqsubseteq \exists r^-.C, C \sqsubseteq \forall r. \forall r.B\}$ .

Exercise 2.4. Lukas wants to apply the tableau algorithm for checking the satisfiability of the concept  $B \sqcap \exists r^-.A$  w.r.t. the TBox  $\{A \sqsubseteq \exists r^-.A \sqcap \exists r.B, \top \sqsubseteq \leqslant 1 r\}$ . He arrives at the situation depicted below and concludes that no further rules are applicable, since  $v_2$  is blocked by  $v_1$ . What is Lukas' error? Continue the algorithm until its termination. (You don't have to illustrate all intermediate steps, just provide the final state.)

$$v_{0}$$

$$r^{-} \downarrow \qquad \qquad L(v_{0}) = \{B \sqcap \exists r^{-}.A, B, \exists r^{-}.A, C_{\mathcal{T}}, \neg A, \leqslant 1 r\}$$

$$r^{-} \downarrow \qquad \qquad L(v_{1}) = \{A, C_{\mathcal{T}}, \exists r^{-}.A, \exists r.B, \leqslant 1 r\}$$

$$L(v_{2}) = \{A, C_{\mathcal{T}}, \exists r^{-}.A, \exists r.B, \leqslant 1 r\}.$$

**Exercise 2.5.** Extend the  $\leq 1$  rule in a way that also qualified functionality axioms of the form  $\top \sqsubseteq \leq 1$  r.A can be treated correctly, where A is an atomic concept. Can you also treat arbitrary axioms of the form  $C \sqsubseteq \leq 1$  r.D?