Prof. Dr. Sebastian Rudolph

Formal Concept Analysis Exercise Sheet 2, Winter Semester 2015/16

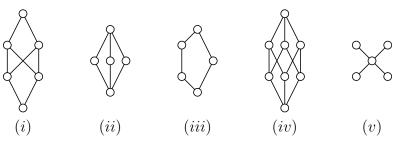
1 Lattice Theory

Exercise 1 (line diagram)

a) Define: What is a lattice?

b) Find a preferably small lattice and draw its line diagram.

c) Which of the following line diagrams does not represent a lattice? Why?



Exercise 2 (complete lattice)

a) Define: What is a complete lattice?

b) Can you find a *complete* lattice among the lattices of Exercise 1c?

c) Let $P := (M, \leq)$ be an ordered set such that for every subset X of M the infimum $\bigwedge X$ exists. Show that P is a complete lattice.

Exercise 3

Prove the following theorem:

Let (L, \leq) be a lattice with supremum and infimum defined as usual. For any elements $x, y, z \in$ L holds:

(i)
$$x \wedge y = y \wedge x$$

(ii)
$$x \lor y = y \lor x$$

(iii)
$$x \wedge (y \wedge z) = (x \wedge y) \wedge z$$
 (iv) $x \vee (y \vee z) = (x \vee y) \vee z$

(iv)
$$x \vee (y \vee z) = (x \vee y) \vee z$$

(v)
$$x \wedge (x \vee y) = x$$

(vi)
$$x \lor (x \land y) = x$$

(vii)
$$x \wedge x = x$$

(viii)
$$x \lor x = x$$

Exercise 4 (the first formal concepts)

Try to compute all formal concepts of the formal context shown in Table 1.

Tabelle 1: Grobian Gans: $Die\ Ducks.\ Psychogramm\ einer\ Sippe.$ Rowohlt, Reinbek bei Hamburg 1972, ISBN 3-499-11481-X

	generation			sex		financial status		
	older	middle	younger	male	female	rich	carefree	indebted
Tick			×	×			×	
Trick			×	×			×	
Track			×	×			×	
Donald		×		×				×
Daisy		×			×		×	
Gustav		×		×			×	
Dagobert	×			×		×		
Annette	×				×		×	
Primus v. Quack	×			×			×	