PROBLEM SOLVING AND SEARCH IN ARTIFICIAL INTELLIGENCE

Lecture 9 Evolutionary Algorithms

Sarah Gaggl

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DRESDEN concept
Exzellenz aus Wissenschaft und Kultur

Agenda

1. Introduction
2. Constraint Satisfaction Problems (CSP)
3. Uninformed Search versus Informed Search (Best First Search, A* Search, Heuristics)
4. Local Search, Stochastic Hill Climbing, Simulated Annealing
5. Tabu Search
6. Answer-set Programming (ASP)
7. Evolutionary Algorithms/ Genetic Algorithms
8. Structural Decomposition Techniques (Tree/Hypertree Decompositions)

Outline

1. Motivation
2. Structure of EAs
3. Components of EAs
4. Working of EAs
5. Conclusion

Motivation

- Search algorithms so far modified (resp. constructed) one single solution.
- Process a complete solution or construct the final solution from smaller building blocks.
- There is a single best solution to be improved.

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New Idea

- Work on a population of solutions
- Let the solutions compete against each other
- Use random variation to search for new solutions

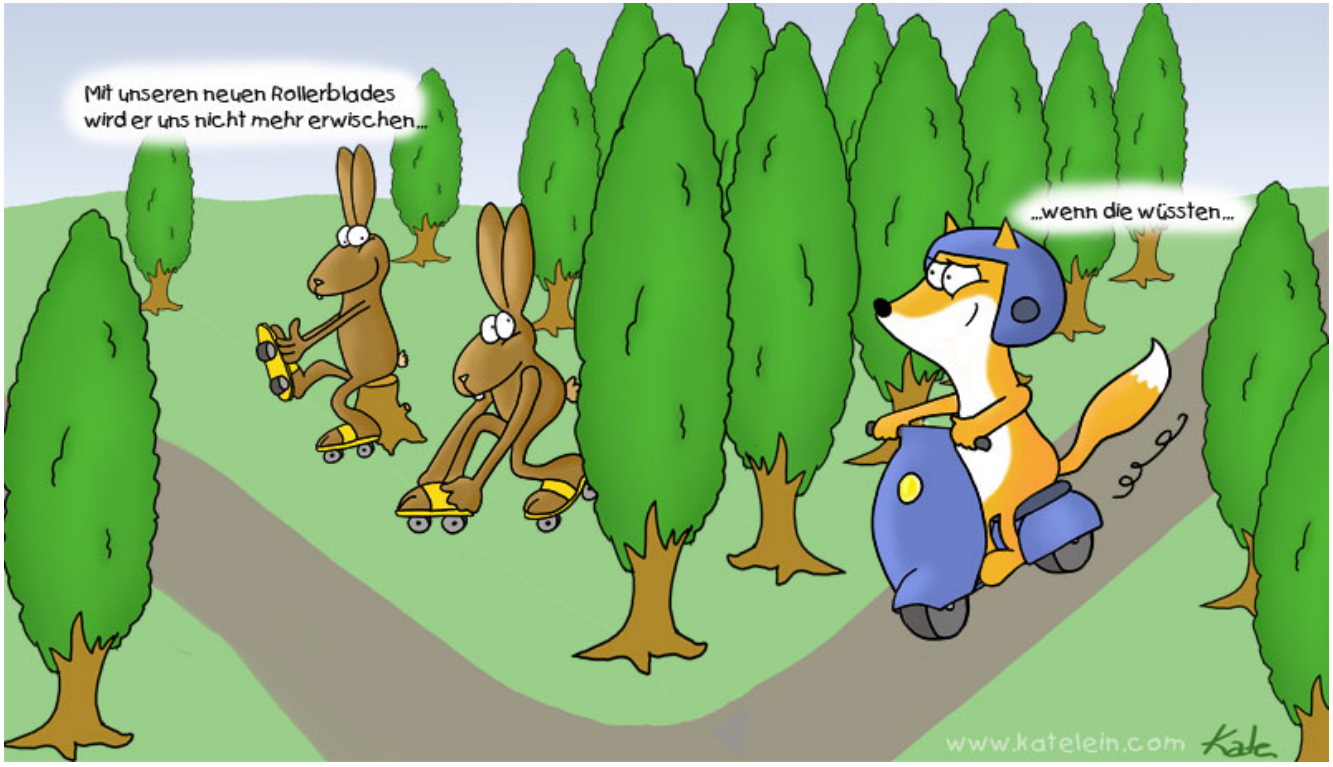
Rabbits and Foxes

- Some rabbits are faster and smarter - they don't get eaten by foxes
- They do what rabbits do best: make more rabbits
- Breeding mixes the rabbits' genetic material
- Every once in a while: mutation
- Over generations, rabbits become faster and smarter

Rabbits and Foxes ctd.

- The same happens with foxes
- They are forced to get better at finding a meal

Rabbits and Foxes ctd.



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Evolutionary Algorithms (EAs)

- A population of individuals exists in an environment with limited resources
- Competition for resources causes selection of fitter individuals that are better adapted to environment
- These individuals act as seeds for generation of new individuals through recombination and mutation
- New individuals have their fitness evaluated and compete (possibly also with parents) for survival
- Over time natural selection causes a rise in the fitness of the population

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Facts on EAs

- EAs are generate and test algorithms
- They are stochastic and population-based
- Variation operators (recombination and mutation) create necessary diversity
- Selection reduces diversity and acts as a force pushing quality

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2. Structure of EAs
   - General Schema
   - Pseudo-Code
3. Components of EAs
   - Representation
   - Evaluation Function
   - Population
   - Parent Selection Mechanism
   - Variation Operators
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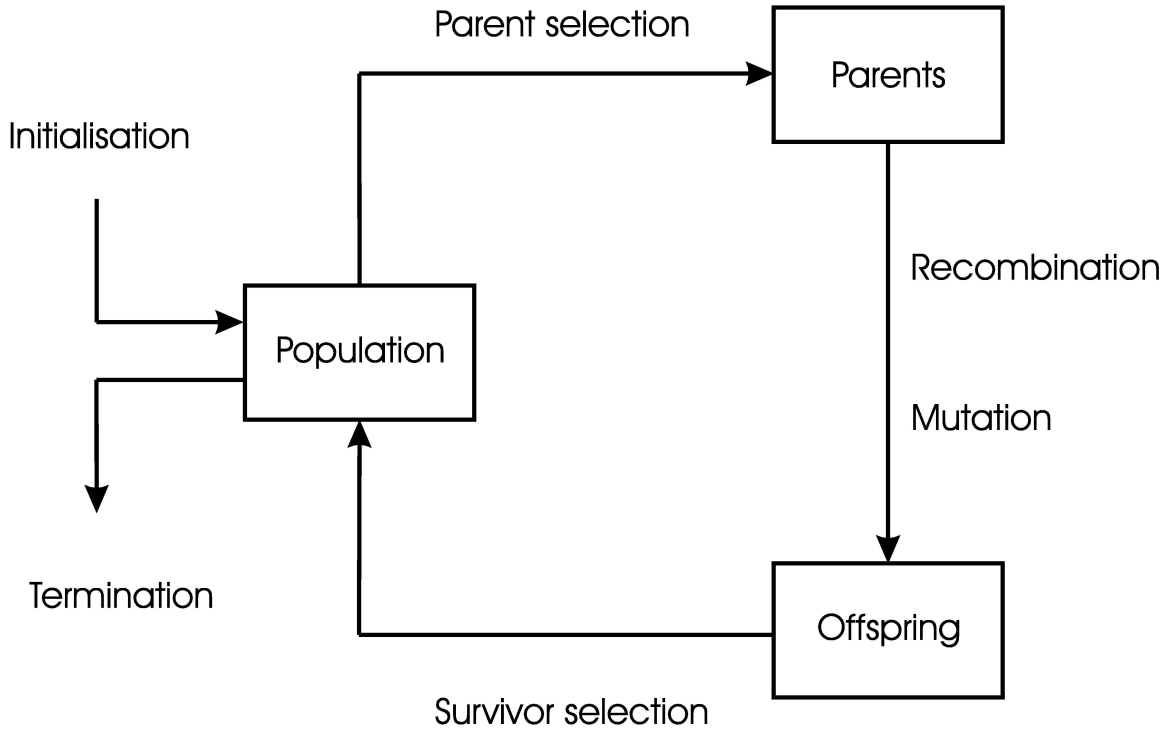
General Schema of EAs



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Structure of an EA

Algorithm 1: evolutionary algorithm

INITIALISE population with random candidate solutions
EVALUATE each candidate
while not TERMINATION-CONDITION is satisfied **do**
  SELECT parents
  RECOMBINE pairs of parents
  MUTATE the resulting offspring
  EVALUATE new candidates
  SELECT individuals for the next generation
end while

Types of EAs

Historically different types of EAs have been associated with different representations.

- Binary strings: Genetic Algorithms
- Real-valued vectors: Evolution Strategies
- Finite state machines: Evolutionary Programming
- Trees: Genetic Programming

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Technically

- choose representation to suit problem
- choose variation operators to suit representation
- selection operators only use fitness – so they are independent of representation

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Components of EAs

- Representation (definition of individuals)
- Evaluation function (or fitness function)
- Population
- Parent selection mechanism
- Variation operators, recombination and mutation
- Survivor selection mechanism (replacement)

Necessary

- Initialisation procedure
- Termination condition

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Representation

- Candidate solutions (individuals) exist in phenotype space
- They are encoded in chromosomes, which exist in genotype space
  - Encoding: phenotype $\rightarrow$ genotype (not necessarily one-to-one)
  - Decoding: genotype $\rightarrow$ phenotype (mutst be one-to-one)
- Chromosomes contain genes, which are in (usually fixed) positions called loci (sing. locus) and have a value (allele)

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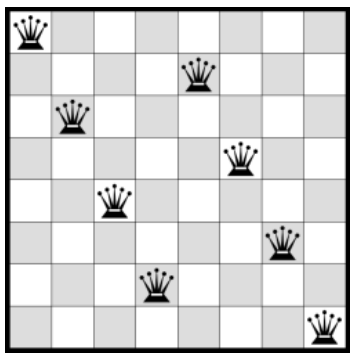
Example (8-Queens)



1	3	5	7	2	4	6	8

- Phenotype: a board configuration
- Genotype: a permutation of the numbers $1 - 8$

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Evaluation (Fitness) Function

- Represents the requirements the population should adopt to
- aka quality function or objective function
- Assigns a single real-valued fitness to each phenotype which forms the basis for selection
- Typically we talk about fitness being maximized

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Example (8-Queens ctd.)

- Penalty of one queen: number of queens she can check
- Penalty of a configuration: sum of penalties of all queens

$\Rightarrow$ Penalty needs to be minimized

$\Rightarrow$ Fitness of a configuration: inverse penalty to be maximized

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Population

- Holds (representations of) possible solutions
- Usually has a fixed size and is a multiset of genotypes
- Some sophisticated EAs also assert a spatial structure on the population e.g. a grid
- Selection operators usually take whole population into account i.e. reproductive probabilities are relative to current generation
- Diversity of a population refers to the number of different solutions
- No single measure for diversity exists
- Typically one refers to number of different fitness values/phenotypes/genotypes present

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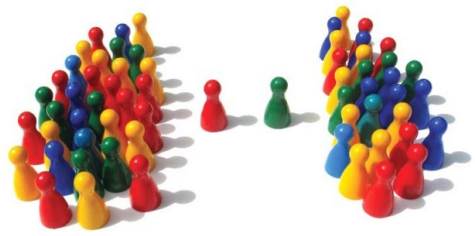
Parent Selection Mechanism



- Distinguish among individuals based on their quality – allow better individuals to become parents of next generation
- Individual is a parent if it has been selected to create offspring
- Responsible for pushing quality improvements
- Usually probabilistic
  - high quality solutions more likely to become parents than low quality
  - BUT: not guaranteed
  - even worst in current population has non-zero probability of becoming a parent
- Stochastic nature can aid escape from local optima

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Example (8-Queens ctd.)

Pick 5 parents randomly and take the two best to generate offspring.

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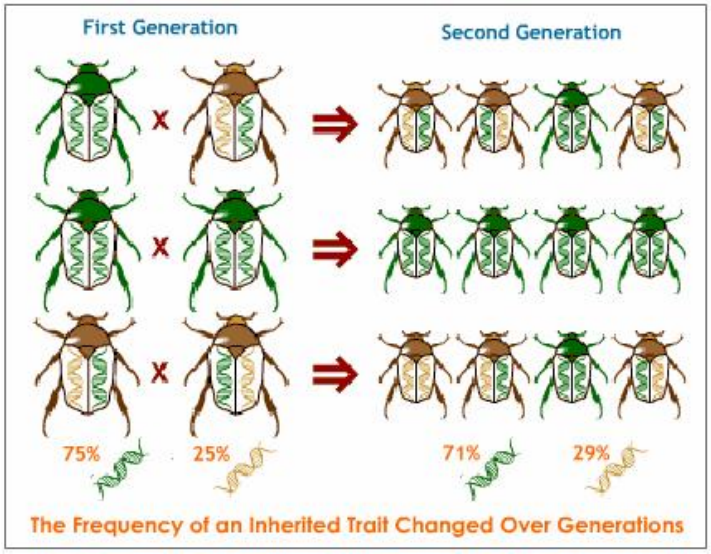
Variation Operators

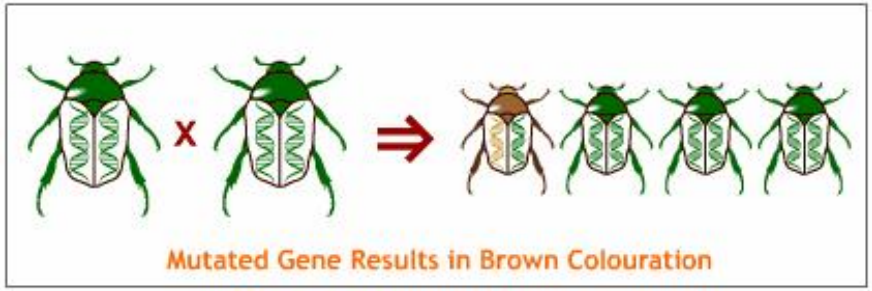
- Role is to generate new candidate solutions
- Usually divided into two types according to their arity

Arity 1: mutation operator
Arity >1: recombination operator; arity=2 typically called crossover

- Debate about relative importance of recombination and mutation
  - Nowadays most EAs use both
  - Choice of particular variation operator is representation dependent



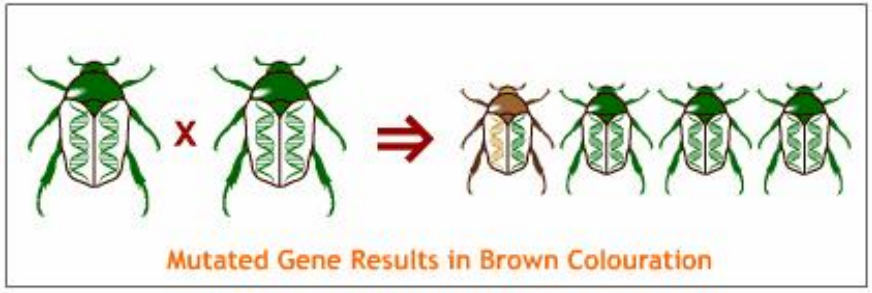
Mutation



- Acts on one genotype and delivers another
- Element of randomness is essential and differentiates if from other unary heuristic operators
- Generating a child amounts to stepping to a new point in search space

$\Rightarrow$ Mutation may guarantee connectedness of search space

Mutation



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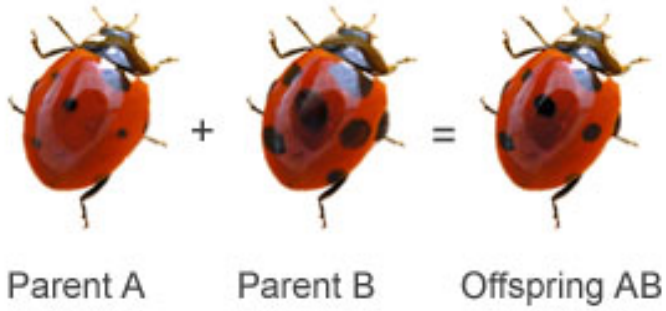
Example (8-Queens ctd.)

Swap values of two randomly chosen positions

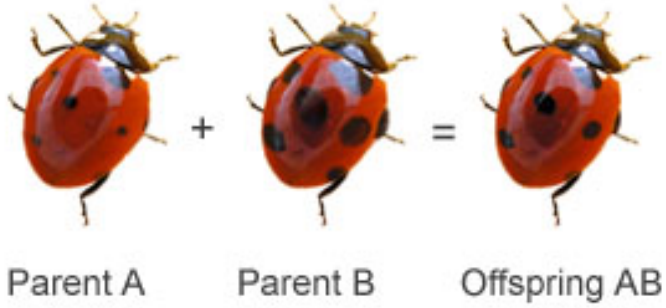
1	3	5	7	2	4	6	8	$\Rightarrow$	1	3	6	7	2	4	5	8

Recombination



- Merges information from parents into offspring
- Choice of what information to merge is stochastic
- Most offspring may be worse, or the same as parents
- Hope that some are better by combining elements of genotypes that lead to good traits

Recombination



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Note

- Variation operators are representation dependent
- Different representations require different variation operators

Example (8-Queens ctd.)

Combine two permutations into two new permutations

- Choose random crossover point
- Copy fist parts into children
- Create second part by inserting values from other parent
  - in order they appear there
  - beginning after crossover point
  - skipping values already in child

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8	7	6	5	4	3	2	1

$\Rightarrow$

1	3	5	4	2	8	7	6
8	7	6	2	4	1	3	5

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$\Rightarrow$

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Children inherit genetic material from both parents!

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Survivor Selection

- aka replacement
- Most EAs use fixed population size
- Often deterministic
  - Fitness-based: e.g. rank parents and offspring and take the best
  - Age-based: make as many offspring as parents and delete all parents
  - Combinations of the former two (elitism)

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Example (8-Queens ctd.)

Merge population and offspring – rank them according to fitness – delete the worst two.

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Initialization/Termination

Initialization

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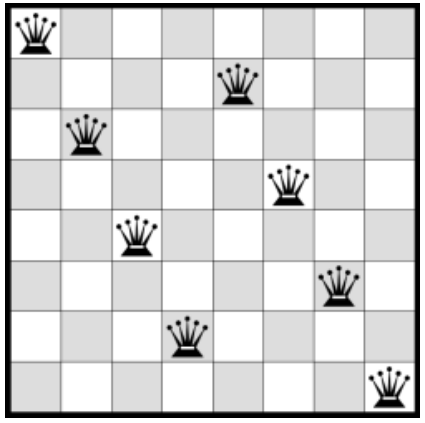
Termination Condition

- Checked every iteration
- Reaching some (known/hoped for) fitness
- Reaching some maximum allowed number of generations
- Reaching some minimum level of diversity
- Reaching some specified number of generations without fitness improvement

Example (8-Queens ctd.)



- Initial population: randomly generated permutations
- Termination condition: solution or 10000 fitness evaluations
- Population size: 100
- Recombination probability: 100%
- Mutation probability: 80%

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Working of EAs

Typical progress of an EA illustrated in terms of population distribution

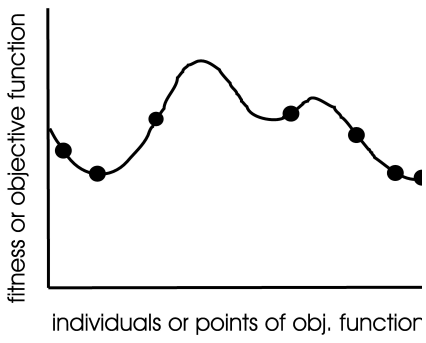


Figure: begin

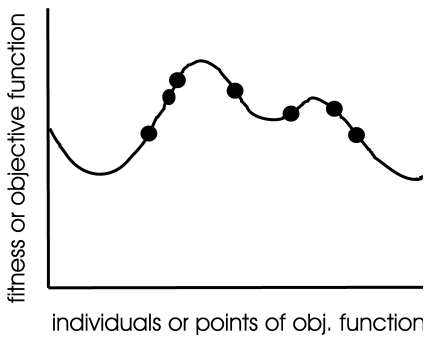


Figure: halfway

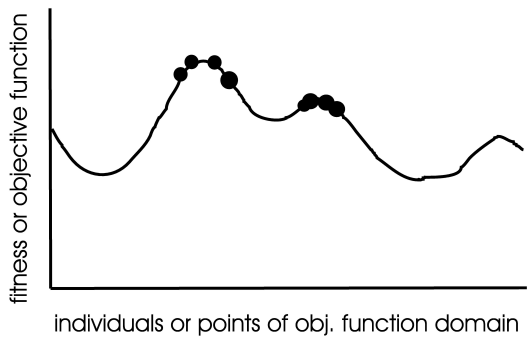
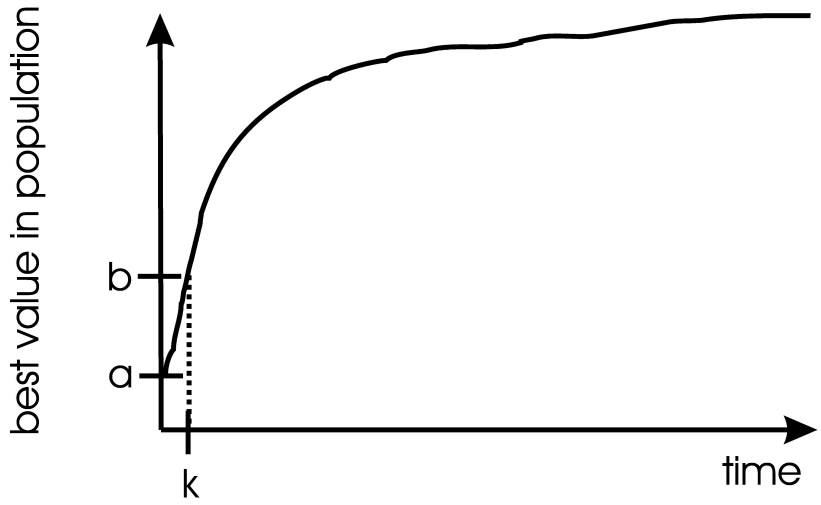


Figure: end

Working of EAs ctd.

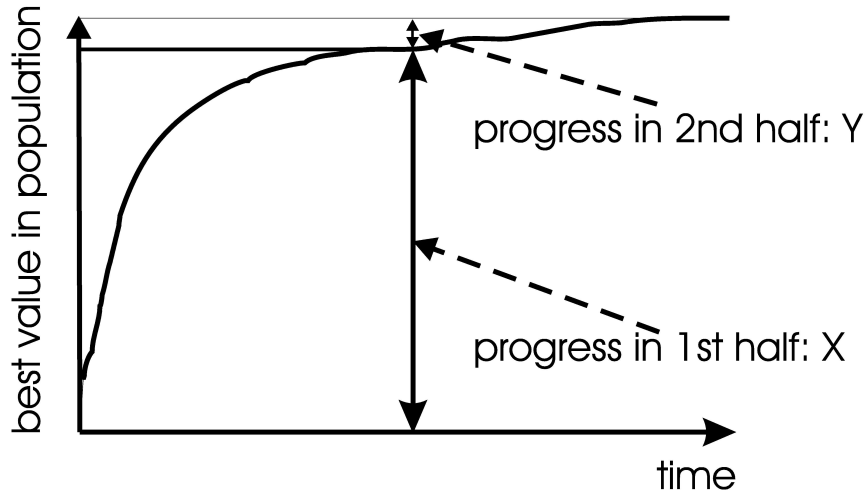
Typical progress in terms of development of best fitness value within population in time.

No Need for Heuristic Initialisation



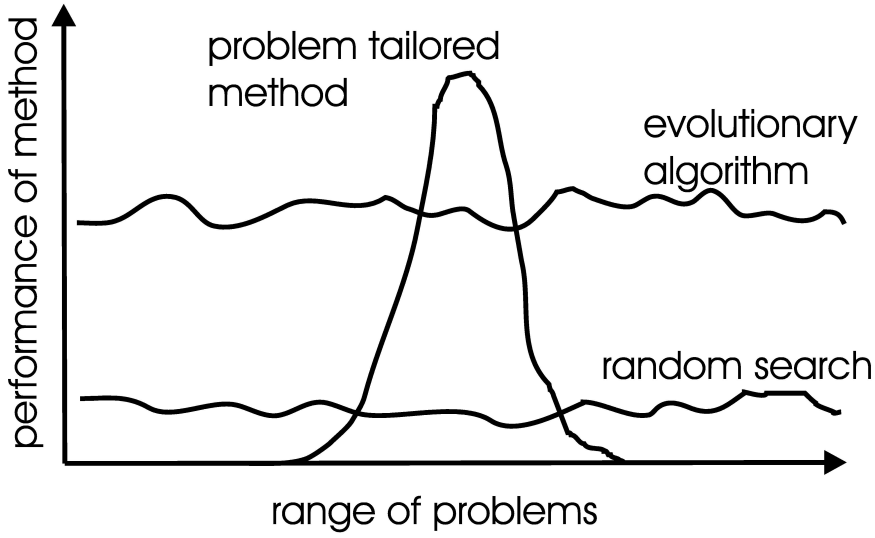
- Level a: best fitness in a randomly initialised population
- Level b: heuristic initialisation
- After k generations same level reached

Termination Conditions



- Divide the run into two equally long sections
- Fitness increases in the first half X
- Progress in second half Y is much smaller
- Due to anytime behaviour, efforts spent after a certain time may not result in better solution quality

Performance from Global Perspective



- Performance on a wide range of problems
- EAs are robust problem solving tools
- For most problems a problem-specific tool may
  - perform better than a generic search algorithm on most instances
  - have limited utility
  - not do well on all instances
- EAs provide an evenly good performance over a range of problems and instances

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Summary

- Idea for EAs come from evolution theory
- Components
  - Representation (definition of individuals)
  - Evaluation function (or fitness function)
  - Population
  - Parent selection mechanism
  - Variation operators, recombination and mutation
  - Survivor selection mechanism (replacement)
  - Initialisation and termination condition
- Performance

References

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