Exercise Sheet 12: Dependencies
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Exercise 12.1. Let $\mathcal{L}$ be a fragment of first-order logic for which finite model entailment and arbitrary model entailment coincide, i.e., for every $\mathcal{L}$-theory $\mathcal{T}$ and every $\mathcal{L}$-formula φ, we find that φ is true in all models of $\mathcal{T}$ if and only if φ is true in all finite models of $\mathcal{T}$.

(a) Give an example for a proper fragment of first-order logic with this property.

(b) Give an example for a proper fragment of first-order logic without this property.

(c) Show that entailment is decidable in any fragment with this property.

Exercise 12.2. Consider the following set of tgds Σ:

$$\begin{aligned} \mathsf{A}(x) &\to \exists y.\ \mathsf{R}(x,y) \wedge \mathsf{B}(y) \\ \mathsf{B}(x) &\to \exists y.\ \mathsf{S}(x,y) \wedge \mathsf{A}(y) \\ \mathsf{R}(x,y) &\to \mathsf{S}(y,x) \\ \mathsf{S}(x,y) &\to \mathsf{R}(y,x) \end{aligned}$$

Does the oblivious chase universally terminate for Σ? What about the restricted chase?

Exercise 12.3. Is the following set of tgds Σ weakly acyclic?

$$\begin{aligned} \mathsf{B}(x) &\to \exists y.\ \mathsf{S}(x,y) \wedge \mathsf{A}(x) \\ \mathsf{A}(x) \wedge \mathsf{C}(x) &\to \exists y.\ \mathsf{R}(x,y) \wedge \mathsf{B}(y) \end{aligned}$$

Does the skolem chase universally terminate for Σ?

Exercise 12.4. Termination of the oblivious (resp. restricted) chase over a set of tgds Σ implies the existence of a finite universal model for Σ. Is the converse true? That is, does the existence of a finite universal model for Σ imply termination of the oblivious (resp. restricted) chase?

Exercise 12.5. Consider a set of tgds Σ that does not contain any constants. A term is *cyclic* if it is of the form $f(t_1, \ldots, t_n)$ and, for some $i \in \{1, \ldots, n\}$, the function symbol f syntactically occurs in t_i. Then Σ is *model-faithful acyclic* (MFA) iff no cyclic term occurs in the skolem chase of $\Sigma \cup \mathcal{I}_\star$, where $\mathcal{I}_\star$ is the critical instance.

Show the following claims:

1. Checking MFA membership is decidable.
2. Is the set of tgds from Exercise 12.3 MFA?
3. If a set of tgds Σ without constants is MFA, then the skolem chose universally terminates for Σ.