

Foundations of Knowledge Representation

Description Logic - Problems 2

Problem 1. Write down the following:

1. An unsatisfiable \mathcal{ALC} -knowledge base \mathcal{K} whose $TBox$ is satisfiable and whose $ABox$ contains only role assertions.
2. An unsatisfiable \mathcal{ALC} -knowledge base whose $TBox$ is empty.
3. An unsatisfiable \mathcal{ALC} - $TBox$.
4. A satisfiable \mathcal{ALC} - $TBox$ \mathcal{T} such that all the atomic concepts occurring in \mathcal{T} are unsatisfiable w.r.t. \mathcal{T} . Write down a model of \mathcal{T} .
5. A satisfiable \mathcal{ALC} -knowledge base such that all its models contain at least two domain individuals.

Hint: For this exercise, the top concept (\top) and the bottom concept (\perp) are of great help. In the first problem, for instance, you could choose as satisfiable $TBox$:

$$\mathcal{T} = \{\exists R.\top \sqsubseteq \perp\}$$

Problem 2. Which of the following statements is true with respect to the empty $TBox$ (that is, all possible interpretations)?

1. $A \sqcap \neg A$ is satisfiable.
2. $A \sqcup \neg A$ is satisfiable.
3. $A \sqcap \exists r.B \sqcap \exists r.\neg B$ is satisfiable.
4. $A \sqcap \exists r.B \sqcap \forall s.\neg B$ is satisfiable.
5. $A \sqcap \exists r.B \sqcap \forall r.\neg B$ is satisfiable.

6. $A \sqcap \neg A$ is subsumed by B .
7. B is subsumed by $A \sqcup \neg A$.
8. $A \sqcap \exists r.B$ is subsumed by $A \sqcap \exists r.\top$.
9. $A \sqcap \forall r.B$ is subsumed by $A \sqcap \exists r.B$.
10. $A \sqcap \exists r.B$ is subsumed by $A \sqcap \forall r.B$.

Problem 3. *Extend the knowledge base you built in Problem 2 from the previous Exercise Sheet to capture the following statements. You may use additional features such as number restrictions if needed; you may also need more than one axiom for some of the statements. Moreover, you are allowed to use the concept name "Broken" to build your extended knowledge base.*

1. *Cars have between three and four wheels.*
2. *Bicycles have exactly two wheels.*
3. *A human who legally controls a car holds a driving license and is an adult (this is a difficult one!).*
4. *A vehicle is controlled by exactly one human.*
5. *A thing's parts' parts are that thing's parts.*
6. *A car with a broken part is broken.*
7. *Bob controls a car with a wheel that has a broken axle.*