Problem 5.1
Consider the following theory:

\[ F = \{ \begin{align*}
0 + X &= X \\
s(X) + Y &= s(X + Y)
\end{align*} \]

1. Show that \( F \not| (\forall)(X + 0 = X) \)
2. Specify a formula \( F' \) such that \( F \cup F' \models (\forall)(X + 0 = X) \)
3. Show that every minimal Herbrand model of \( F \) is a model of the formula \( (\forall)(X + 0 = X) \).
4. Show that every minimal Herbrand model of \( F \) is a model of the formula \( (\forall)(X + Y = Y + X) \).
5. Specify a formula \( P(x) \) such that

\[ F \cup \{(P(0) \land (\forall X)(P(x) \rightarrow P(s(x)))) \rightarrow (\forall X)P(X)\} \models (\forall)(X + Y = Y + X) \]

Problem 5.2
The previous exercise defined the addition on natural numbers.

Specify a Horn-theory that defines the addition on integers.