ATTRIBUTE EXPLORATION

A Brief Introduction

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Attribute Implications (aka propositional Horn clauses)

- For $A, B \subseteq M$, the *implication* $A \rightarrow B$ *holds* in $\mathbb{K}$, if every object having all attributes from A also has all attributes from B.
- Formally: $A \subseteq \{g\}'$ implies $B \subseteq \{g\}'$ for all $g \in G$
- Examples:
  - $\{wet\} \rightarrow \{fluid\}$
  - $\{fluid, dry\} \rightarrow \{warm\}$
  - $\{dry, wet\} \rightarrow \{cold\}$ (!)

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

How to „Datamine“ Implications?

- We want to extract the „implicational“ knowledge from a formal context.
- Very naive approach: enumerate all $(2^{2|M|})$ implications and check against context.
  - Takes way too long.
  - Generated implication set is extremely redundant.
- Examples:
  - $\{\text{fluid}, \text{dry}\} \to \{\text{fluid}\}$
  - $\{\text{wet}\} \to \{\text{fluid}\}$ vs. $\{\text{wet}, \text{cold}\} \to \{\text{fluid}\}$

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

How to „Datamine“ Implications?

- Observations:
  - For any attribute set A, the implication $A \rightarrow A''$ holds in $\mathbb{K}$
  - If $A \rightarrow B$ holds in $\mathbb{K}$ then $B \subseteq A''$
- Hence the implications of the form $A \rightarrow A''$ provide enough information about all implications of the context.
- Still rather naive approach: enumerate all ($2^{|M|}$) attribute sets A and generate implication $A \rightarrow A''$
  - Still takes way too long
  - Generated implication set is still extremely redundant

Implication Bases

- Given a formal context $\mathbb{K}$, a set of implications $\Im$ is called *implication base* of $\mathbb{K}$, if ...
  - every implication $A \rightarrow B$ from $\Im$ holds in $\mathbb{K}$,
  - every implication $A \rightarrow B$ holding in $\mathbb{K}$ can be derived from $\Im$, and
  - none of the implications from $\Im$ can be derived from the other implications contained in $\Im$
- Question: which $A \rightarrow A''$ to choose to make up an implication base?

The Stem Base

- Question: which $A \rightarrow A''$ to choose to make up an implication base?
- Answer: take all the *pseudo-intents* of $\mathbb{K}$.
- Attribute set P is called pseudo-intent, if
  - P is not an intent (i.e. $P \neq P''$), but
  - if P contains another pseudo-intent Q, then it also contains Q''
- Definition recursive (but OK at least for finite M)
- Set $\{P \rightarrow P'' \mid P \text{ pseudo-intent}\}$ is called *stem base*

How to Compute the Stem Base

- We order attributes in a row:
  - e.g. a,b,c,d,e,f
- Based on that order, attribute sets are encoded as bit-vectors of length |M|
  - e.g. $\{a,c,d\}$ becomes [1,0,1,1,0,0]
- Implications are pairs of bit-vectors
  - e.g. $\{a\} \to \{a,e,f\}$ becomes ([1,0,0,0,0,0], [1,0,0,0,1,1])
- Implications can be „applied" to attribute sets
  - ({a} → {a,e,f}) applied to $\{a,c,d\}$ yields $\{a,c,d,e,f\}$
    ([1,0,0,0,0,0], [1,0,0,0,1,1]) [1,0,1,1,0,0] = [1,0,1,1,1,1]
- Implication sets can be applied to attribute sets:
  - {{b,d}→{c},{a}→{d}} applied to {a,b} yields {a,b,c,d}
  - write $\mathfrak{I}$(A) for the result of applying implication set $\mathfrak{I}$ to attribute set A
- A+i defined as: take A, set ith bit to 1 and all subsequent bits to 0
  - e.g. [0,1,0,0,1,1]+3 = [0,1,1,0,0,0]

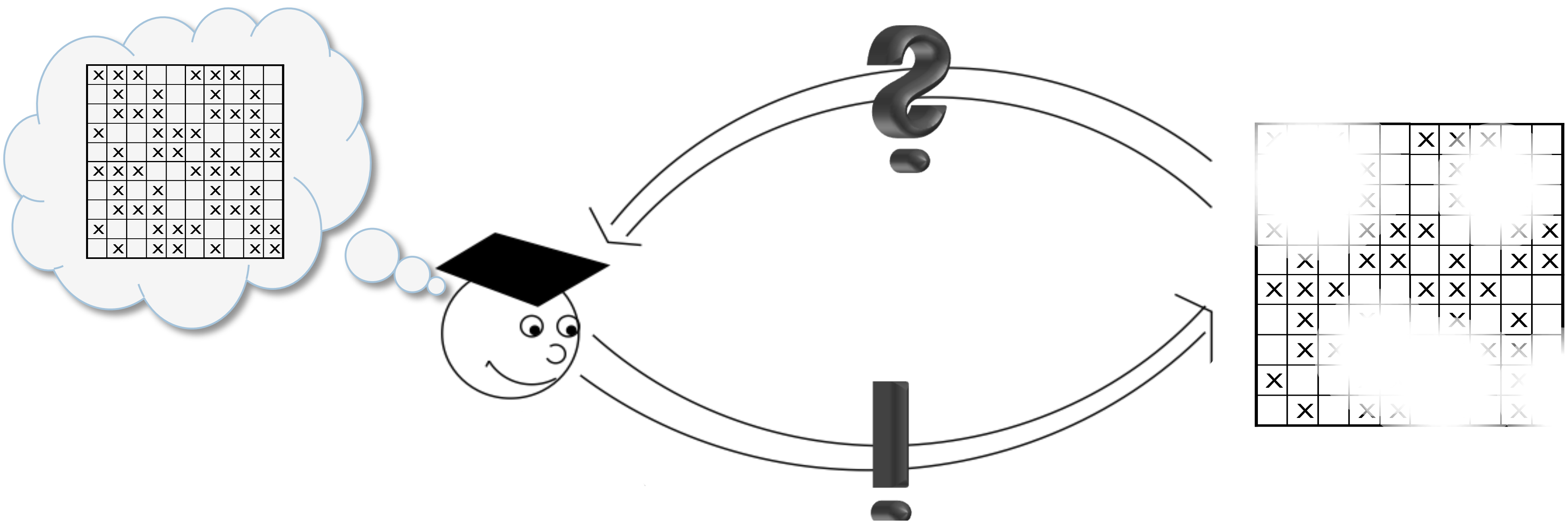
How to Compute the Stem Base

- Input formal context $\mathbb{K}$
- Create list $\mathfrak{I}$ of implications, initially empty
  Let A = [0,0,...,0] *(bit representation of empty set)*
- Repeat
  - Add A → A'' to $\mathfrak{I}$ in case A ≠ A''
  - Starting from i = |M|+1, decrement i until
    - i=0 or
    - The ith bit of A is 0 and
      applying $\mathfrak{I}$ to A+i produces 1s only at positions greater than i
  - If i=0 output $\mathfrak{I}$ and exit
  - Let A = $\mathfrak{I}$(A+i)

... i ...
A: [0,0,1,0,1,1,0]
A+i: [0,0,1,1,0,0,0]
$\mathfrak{I}$(A+i): [0,0,1,1,0,1,1]

Interactive Knowledge Acquisition via Attribute Exploration

- Sometimes, $\mathbb{K}$ is not entirely known from the beginning, but implicitly present as an expert‘s knowledge
- Attribute exploration determines the stembase of $\mathbb{K}$ by asking expert for missing information



Interactive Knowledge Acquisition via Attribute Exploration

- Sometimes, $\mathbb{K}$ is not entirely known from the beginning, but implicitly present as an expert‘s knowledge
- Attribute exploration determines the stembase of $\mathbb{K}$ by asking expert for missing information
  - M known and fixed
  - $H \subseteq G$ objects that are known in advance (as well as their attributes)
- Idea: use stembase algorithm on incomplete context which is updated on the fly

Stem Base Algorithm Revisited

- Input formal context $\mathbb{K}=(H,M,J)$ where $J=(H\times M)\cap I$
- Create list $\Im$ of implications, initially empty
  Let $A = [0,0,\ldots,0]$ *(bit representation of empty set)*
- Repeat
  - Add $A \rightarrow A''$ to $\Im$ in case $A \neq A''$
  - Starting from $i = |M|+1$, decrement
    - $i=0$ or
    - The ith bit of A is 0 and applying $\Im$ to $A+i$ produces 1s only at
  - If $i=0$ output $\Im$ and exit
  - Let $A = \Im(A+i)$

Has to be altered, because implication valid in $\mathbb{K}$ might be invalid in $\mathbb{K}$ since refuted by an object not yet recorded. Then augmenting $\mathbb{K}$ by this object allows to refine the hypothesis.

Making It Interactive...

- Instead of just adding $A \to A''$ to $\Im$, do the following control Loop:
  - While $A \neq A''$
    - Ask expert whether $A \to A''$ is valid in $\mathbb{K}$
    - If yes, add $A \to A''$ to $\Im$ and exit while-loop, otherwise ask for counterexample and add it to $\underline{\mathbb{K}}$
- What is a counterexample for $A \to A''$?
  - An object having all attributes from A but missing some from A''
- How to add a counterexample g to $\underline{\mathbb{K}}=(H,M,J)$?
  - $H_{new} = H \cup \{g\}$
  - $J_{new} = J \cup \{(g,m) \mid m \text{ is attribute of } g \text{ in } \mathbb{K}\}$
  - Essentially: just add a line to the cross table

Making It Interactive...

- Instead of just adding $A \to A''$ to $\Im$, do the following control loop:
  - While $A \neq A'$,
    - Ask expert whether $A \to A''$ is valid in $\mathbb{K}$
    - If yes, add $A \to A''$ to $\Im$ and exit while-loop, otherwise ask for counterexample g and add it to $\underline{\mathbb{K}}$
- Remarks:
  - Attribute set of g has to comply with the implications already confirmed
  - Changing $\underline{\mathbb{K}}$ changes the operator $(.)''$
  - It is not obvious (but has to be proven) that this indeed works, i.e. the enumeration done beforehand is not corrupted by updating the context

Stem Base Algorithm Revisited

- Input formal context $\mathbb{K}$
- Create list $\Im$ of implications, initially empty
  Let A = [0,0,...,0]   *(bit representation of empty set)*
- Repeat
  - While A ≠ A',
    - Ask expert whether $A \rightarrow A''$ is valid in $\mathbb{K}$
    - If yes, add $A \rightarrow A''$ to $\Im$ and exit while-loop,
      otherwise ask for counterexample g and add it to $\underline{\mathbb{K}}$
  - Starting from i = |M|+1, decrement i until
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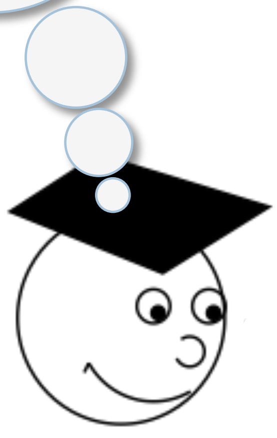
      applying $\Im$ to A+i produces 1s only at positions greater than i
  - If i=0 output $\Im$ and exit
  - Let A = $\Im$(A+i)

A Tiny Example: the Four Elements

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	



A: [0, 0, 0, 0, 0]

A'': [0, 0, 0, 0, 1]

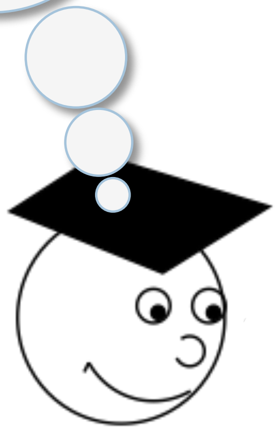
$\{\} \rightarrow \{\text{cold}\}$?

(are all elements cold?)

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×

A Tiny Example: the Four Elements

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	



A: [0, 0, 0, 0, 0]

A'': [0, 0, 0, 0, 1]

$\{\} \rightarrow \{\text{cold}\}$?

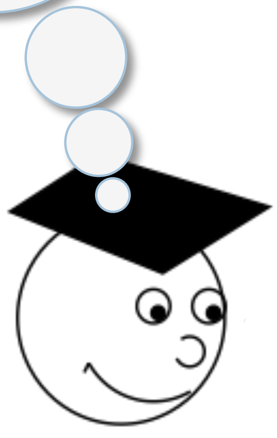
(are all elements cold?)

no: air is not cold!

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	

A Tiny Example: the Four Elements

𝕂	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	



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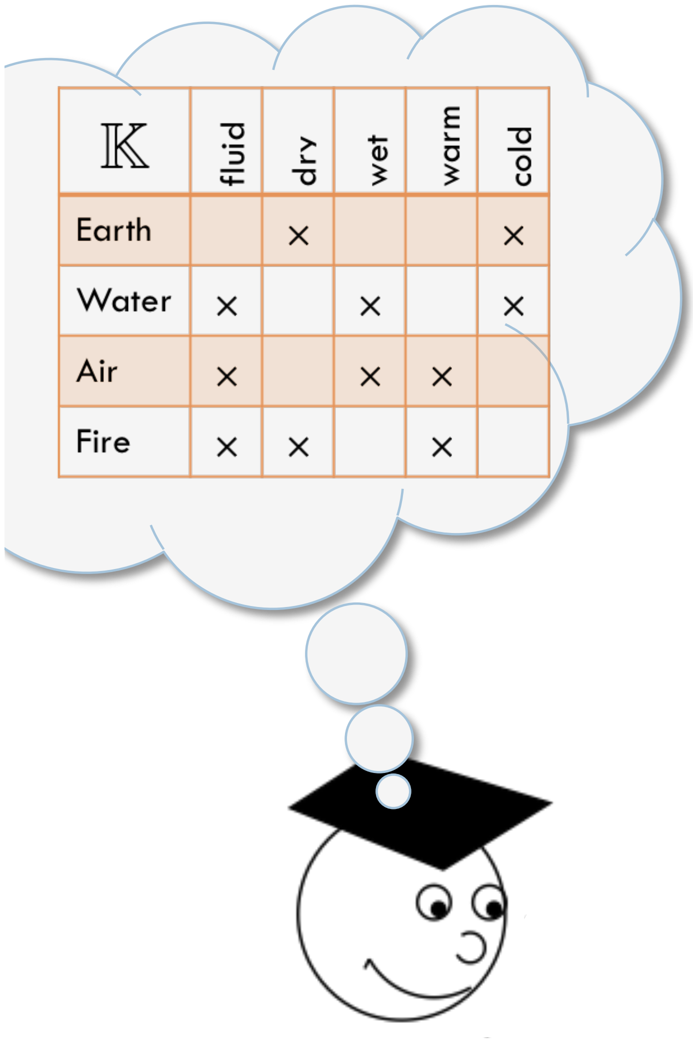
A'': [0, 0, 0, 0, 0]

𝕂	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	

A Tiny Example: the Four Elements



$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

A: [0, 0, 0, 0, 1]

A'': [0, 0, 0, 0, 1]

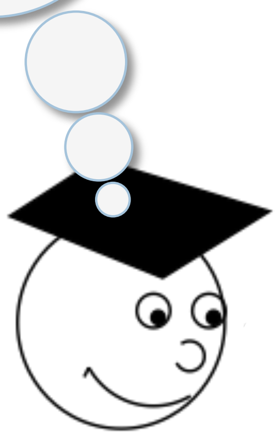
$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	

A Tiny Example: the Four Elements

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

A: [0, 0, 0, 1, 0]

A'': [1, 0, 1, 1, 0]

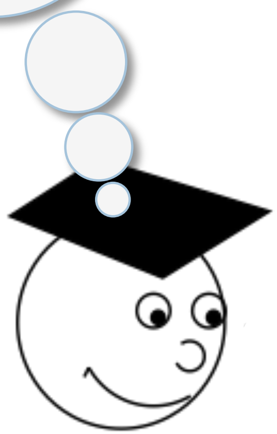


{warm} → {wet,fluid}?

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	

A Tiny Example: the Four Elements

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	



A: [0, 0, 0, 1, 0]

A'': [1, 0, 1, 1, 0]

{warm} → {wet,fluid}?

no: fire is warm but not wet!

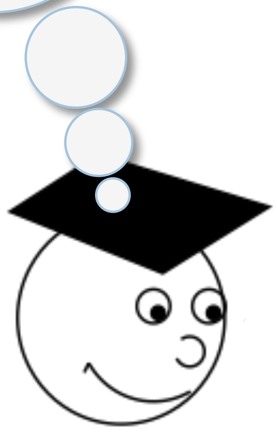
$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

A Tiny Example: the Four Elements

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

A: [0, 0, 0, 1, 0]

A'': [1, 0, 0, 1, 0]



{warm} → {fluid}?

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

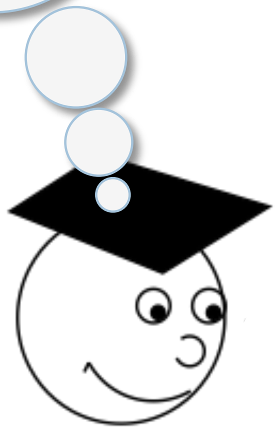
A Tiny Example: the Four Elements

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

[0,0,0,1,0]→[1,0,0,1,0]

A: [0, 0, 0, 1, 0]

A'': [1, 0, 0, 1, 0]



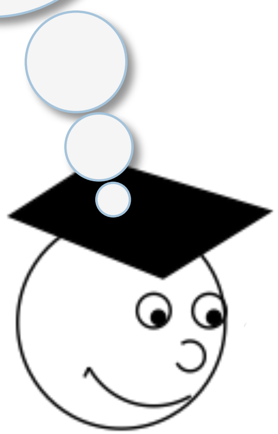
{warm} → {fluid}?

yes!

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

A Tiny Example: the Four Elements

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	



[0,0,0,1,0]→[1,0,0,1,0]

[0,0,1,0,0]→[1,0,1,0,0]

A: [0, 0, 1, 0, 0]

A'': [1, 0, 1, 0, 0]

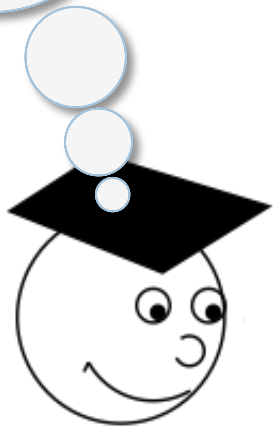
{wet} → {fluid}?

yes!

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

A Tiny Example: the Four Elements

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	



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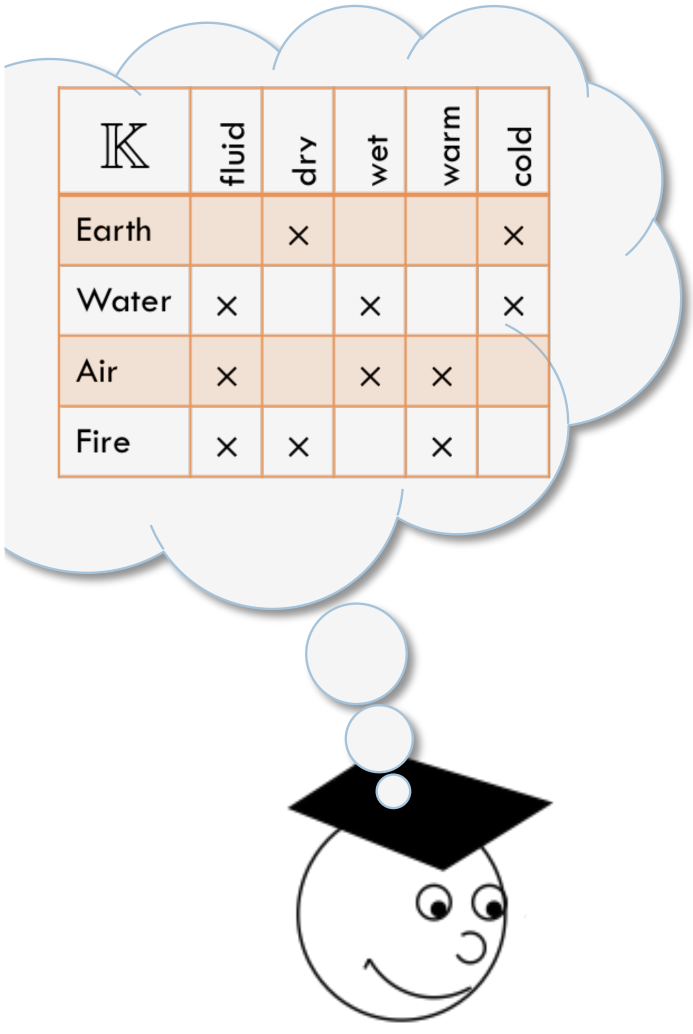
A: [0, 1, 0, 0, 0]

A'': [0, 1, 0, 0, 0]

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

A Tiny Example: the Four Elements



$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

[0,0,0,1,0]→[1,0,0,1,0]

[0,0,1,0,0]→[1,0,1,0,0]

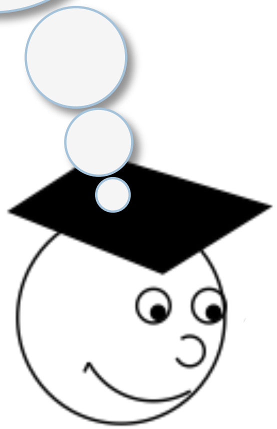
A: [0, 1, 0, 0, 1]

A'': [0, 1, 0, 0, 1]

$\underline{\mathbb{K}}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

A Tiny Example: the Four Elements

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	



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[0,0,1,0,0]→[1,0,1,0,0]

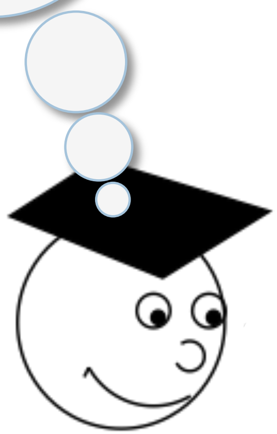
A: [1, 0, 0, 0, 0]

A'': [1, 0, 0, 0, 0]

$\underline{\mathbb{K}}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

A Tiny Example: the Four Elements

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	



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[0,0,1,0,0]→[1,0,1,0,0]

[1,0,0,0,1]→[1,0,1,0,1]

A: [1, 0, 0, 0, 1]

A'': [1, 0, 1, 0, 1]

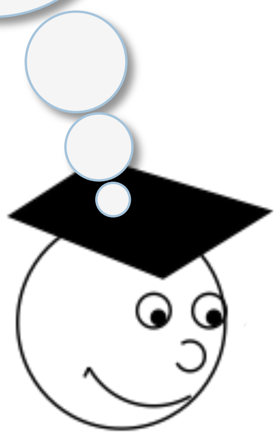
{fluid,cold} → {wet}?

yes!

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

A Tiny Example: the Four Elements

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	



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[1,0,0,0,1]→[1,0,1,0,1]

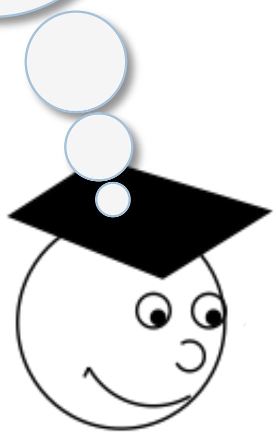
A: [1, 0, 0, 1, 0]

A'': [1, 0, 0, 1, 0]

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

A Tiny Example: the Four Elements

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	



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[0,0,1,0,0]→[1,0,1,0,0]

[1,0,0,0,1]→[1,0,1,0,1]

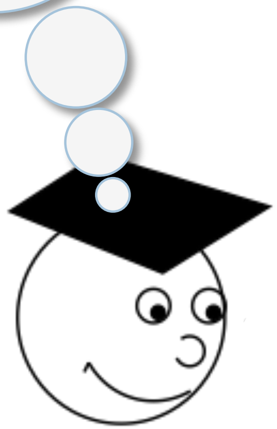
A: [1, 0, 1, 0, 0]

A'': [1, 0, 1, 0, 0]

$\underline{\mathbb{K}}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

A Tiny Example: the Four Elements

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	



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[0,0,1,0,0]→[1,0,1,0,0]

[1,0,0,0,1]→[1,0,1,0,1]

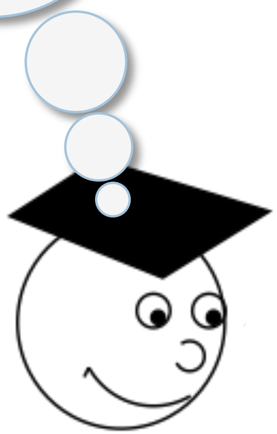
A: [1, 0, 1, 0, 1]

A'': [1, 0, 1, 0, 1]

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

A Tiny Example: the Four Elements

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	



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[1,0,0,0,1]→[1,0,1,0,1]

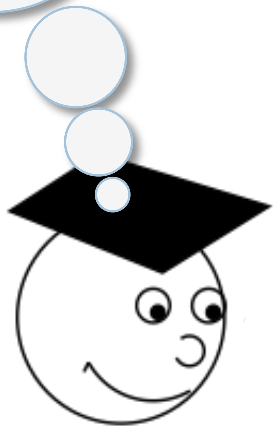
A: [1, 0, 1, 1, 0]

A'': [1, 0, 1, 1, 0]

$\underline{\mathbb{K}}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

A Tiny Example: the Four Elements

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	



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[0,0,1,0,0]→[1,0,1,0,0]

[1,0,0,0,1]→[1,0,1,0,1]

[1,0,1,1,1]→[1,1,1,1,1]

A: [1, 0, 1, 1, 1]

A'': [1, 1, 1, 1, 1]

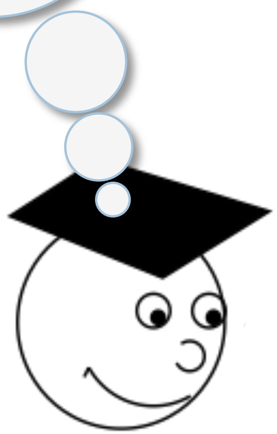
{fluid,wet,warm,cold}
→ everything?

yes!

$\underline{\mathbb{K}}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

A Tiny Example: the Four Elements

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	



{fluid,dry} → {warm}?

A: [1, 1, 0, 0, 0]

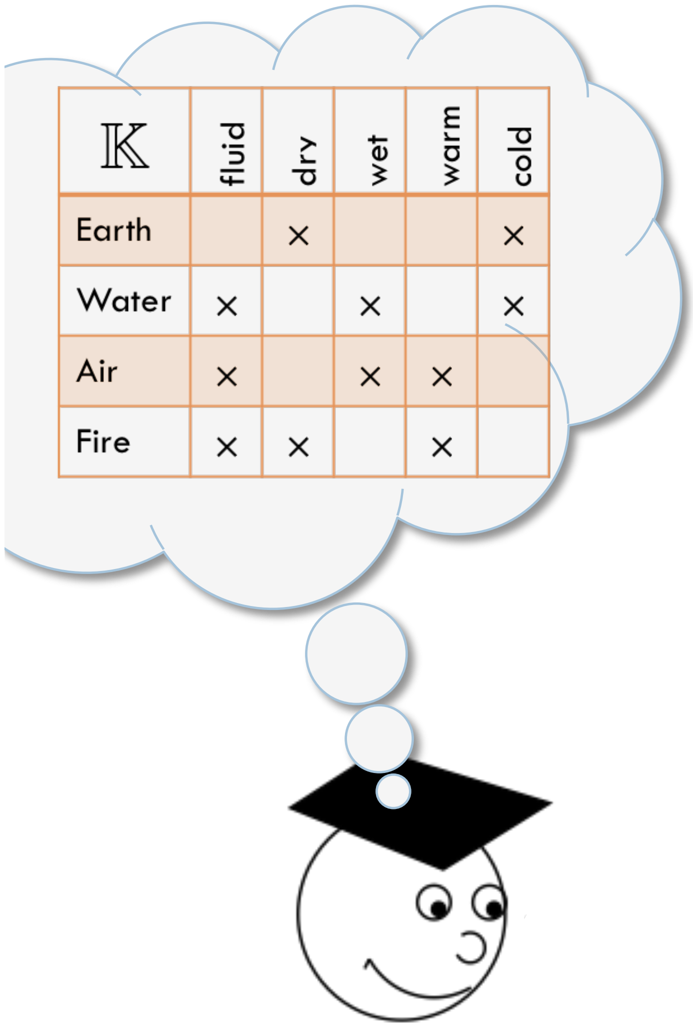
A'': [1, 1, 0, 1, 0]

[0,0,0,1,0]→[1,0,0,1,0]
[0,0,1,0,0]→[1,0,1,0,0]
[1,0,0,0,1]→[1,0,1,0,1]
[1,0,1,1,1]→[1,1,1,1,1]
[1,1,0,0,0]→[1,1,0,1,0]

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

yes!

A Tiny Example: the Four Elements



𝕂	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

A: [1, 1, 0, 1, 0]

A'': [1, 1, 0, 1, 0]

[0,0,0,1,0]→[1,0,0,1,0]

[0,0,1,0,0]→[1,0,1,0,0]

[1,0,0,0,1]→[1,0,1,0,1]

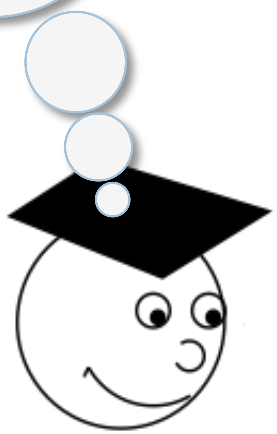
[1,0,1,1,1]→[1,1,1,1,1]

[1,1,0,0,0]→[1,1,0,1,0]

𝕂	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

A Tiny Example: the Four Elements

𝕂	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	



[0,0,0,1,0]→[1,0,0,1,0]

[0,0,1,0,0]→[1,0,1,0,0]

[1,0,0,0,1]→[1,0,1,0,1]

[1,0,1,1,1]→[1,1,1,1,1]

[1,1,0,0,0]→[1,1,0,1,0]

[1,1,0,1,1]→[1,1,1,1,1]

A: [1, 1, 0, 1, 1]

A'': [1, 1, 1, 1, 1]

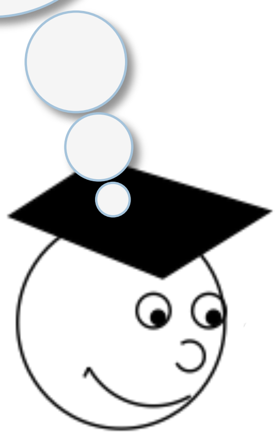
{fluid,dry,warm,cold}
→ everything?

yes!

𝕂	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

A Tiny Example: the Four Elements

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	



[0,0,0,1,0]→[1,0,0,1,0]

[0,0,1,0,0]→[1,0,1,0,0]

[1,0,0,0,1]→[1,0,1,0,1]

[1,0,1,1,1]→[1,1,1,1,1]

[1,1,0,0,0]→[1,1,0,1,0]

[1,1,0,1,1]→[1,1,1,1,1]

[1,1,1,1,0]→[1,1,1,1,1]

A: [1, 1, 1, 1, 0]

A'': [1, 1, 1, 1, 1]

{fluid,dry,wet,warm}

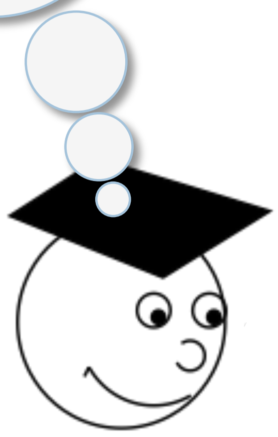
→ everything?

yes!

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

A Tiny Example: the Four Elements

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	



A: [1, 1, 1, 1, 1]

i=0 → terminate

[0,0,0,1,0]→[1,0,0,1,0]

[0,0,1,0,0]→[1,0,1,0,0]

[1,0,0,0,1]→[1,0,1,0,1]

[1,0,1,1,1]→[1,1,1,1,1]

[1,1,0,0,0]→[1,1,0,1,0]

[1,1,0,1,1]→[1,1,1,1,1]

[1,1,1,1,0]→[1,1,1,1,1]

$\mathbb{K}$	fluid	dry	wet	warm	cold
Earth		×			×
Water	×		×		×
Air	×		×	×	
Fire	×	×		×	

Extensions of Classical Attribute Exploration

- Allow for a-priori implications
  - Notion of *relative stem base* (Stumme 1996)
- Allow for arbitrary propositional background knowledge
  - Notion of *frame context* (Ganter 1999)
- Allow for partial description of objects
  - Notion of *partial context* (Burmeister, Holzer 2005)
- Allow for complete exploration of non-propositional logics
  - Horn logic with bounded variables: *rule exploration* (Zickwolff 1991)
  - DLs with bounded role depth: *relational exploration* (Rudolph 2004)

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Formal Concept Analysis

III Knowledge Discovery

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slides based on a lecture by Prof. Gerd Stumme

Agenda

Attribute Exploration

Attribute exploration allows us to compute the stem base interactively, without knowing the context beforehand (or knowing only parts of the context).

We modify the $\textsc{Next Closure}$ algorithm for computing the stem base.

The context can be modified while the list $\mathcal{L}$ of implications is computed by taking into account new objects. If these objects respect all implications that have been computed so far, then the computation can be continued with the results obtained so far. This is the result of the following Lemma:

Lemma: Let $\mathbb{K}$ be a context and let $P_1, P_2, \ldots, P_n$ be the first n pseudo-intents of $\mathbb{K}$ with respect to the lectic order. If $\mathbb{K}$ is extended by an object g the object intent g' of which respects the implications $P_i \to P_i''$, $i \in \{1, \ldots, n\}$, then $P_1, P_2, \ldots, P_n$ are also the lectically first n pseudo-intents of the extended context.

Attribute Exploration

Therefore, if we have found a new pseudo-intent P, we can stop the algorithm and ask, whether the implication $P \to P''$ should be added to $\mathcal{L}$.

The user can answer this question in the affirmative or add a counter-example, which must not contradict the implications he has confirmed so far. In the extreme case, the procedure can be started with a context the object set of which is empty. In this case, the user will have to enter all counter-examples, thereby creating a concept system with a given "attribute logic".

Instead of describing this program in detail, we shall demonstrate its functioning by means of an example: We compute the concept lattice for

$$G = \mathbb{N}$$

$$M = \{\text{even, odd, prime, square, cubic, not prime, not square, not cubic}\}$$

Attribute Exploration: Example

We start with the context

	even	odd	prime	square	cubic	not prime	not square	not cubic
1		×		×	×	×		
2	×		×				×	×
3		×	×				×	×
4	×			×		×		×
5		×	×				×	×
6	×					×	×	×
7		×	×				×	×
8	×				×	×	×	
9		×		×		×		×
27		×			×	×	×	
64	×			×	×	×		

Attribute Exploration: Example

1. suggested implication:

   $$\text{not prime, not square, not cubic} \rightarrow \text{even?}$$

   The answer is "no", since 15 is an odd number that is neither prime nor square nor cubic.

2. suggested implication:

   $$\text{cubic} \rightarrow \text{not prime?}$$

   That is true.

3. suggested implication (all attributes are contained):

   $$\text{cubic, not prime, not cubic} \rightarrow \text{even, odd, prime, square, not square?}$$

   The remark tells the user/expert that all attributes are contained to indicate that the premise could be inconsistent.

Attribute Exploration: Example

4. suggested implication:

$$\text{square} \rightarrow \text{not prime?}$$

   That is true.

5. suggested implication (all attributes are contained):

$$\text{square, not prime, not square} \rightarrow \text{even, odd, prime, cubic, not cubic?}$$

   The implication is accepted because of the attributes in the premise that are negating each other.

6. suggested implication:

$$\text{prime} \rightarrow \text{not square, not cubic?}$$

   This implication also is a property of the natural numbers.

Attribute Exploration: Example

7. suggested implication (all attributes are contained):

   prime, not prime, not square, not cubic → even, odd, square, cubic?

   Here also all attributes appear and the implication is acccepted because of the attributes in the premise that are negating each other.

8. suggested implication (all attributes are contained):

   even, odd → prime, square, cubic, not prime, not square, not cubic?

   This implication is automatically accepted.

Attribute Exploration: Example

Attribute exploration stops here. Seven implications were accepted and one counterexample added to the context. The object set

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 15, 27, 64\}$$

but also the object set

$$\{1, 2, 3, 4, 6, 8, 9, 15, 27, 64\}$$

of the reduced context has the property, that for every non-valid implication there exists at least one counterexample.

Attribute Exploration: Example

The accepted implications, i.e., the stem base, which holds for all natural numbers, looks this way:

1. $\langle 4 \rangle$: cubic $\rightarrow$ not prime
2. $\langle 4 \rangle$: square $\rightarrow$ not prime
3. $\langle 4 \rangle$: prime $\rightarrow$ not square, not cubic
4. $\langle 0 \rangle$: cubic, not cubic $\rightarrow \bot$
5. $\langle 0 \rangle$: square, not square $\rightarrow \bot$
6. $\langle 0 \rangle$: prime, not prime $\rightarrow \bot$
7. $\langle 0 \rangle$: even, odd $\rightarrow \bot$

Attribute Exploration: Example

The corresponding concept lattice. All implications that can be read off hold for *all* natural numbers.