Foundations of Knowledge Representation
Description Logic - Problems 2

Problem 1. *Write down the following:*

1. *An unsatisfiable $\mathcal{ALC}$-knowledge base $\mathcal{K}$ whose TBox is satisfiable and whose ABox contains only role assertions.*

2. *An unsatisfiable $\mathcal{ALC}$-knowledge base whose TBox is empty.*

3. *An unsatisfiable $\mathcal{ALC}$-TBox.*

4. *A satisfiable $\mathcal{ALC}$-TBox $\mathcal{T}$ such that all the atomic concepts occurring in $\mathcal{T}$ are unsatisfiable w.r.t. $\mathcal{T}$. Write down a model of $\mathcal{T}$.*

5. *A satisfiable $\mathcal{ALC}$-knowledge base such that all its models contain at least two domain individuals.*

Hint: For this exercise, the top concept ($\top$) and the bottom concept ($\bot$) are of great help. In the first problem, for instance, you could choose as satisfiable TBox:

$$\mathcal{T} \;=\; \{\exists R.\top \sqsubseteq \bot\}$$

Problem 2. *Which of the following statements is true?*

1. *$A \sqcap \neg A$ is satisfiable.*

2. *$A \sqcup \neg A$ is satisfiable.*

3. *$A \sqcap \exists r.B \sqcap \exists r.\neg B$ is satisfiable.*

4. *$A \sqcap \exists r.B \sqcap \forall s.\neg B$ is satisfiable.*

5. *$A \sqcap \exists r.B \sqcap \forall r.\neg B$ is satisfiable.*

6. $A \sqcap \neg A$ is subsumed by B.

7. B is subsumed by $A \sqcup \neg A$.

8. $A \sqcap \exists r.B$ is subsumed by $A \sqcap \exists r.\top$.

9. $A \sqcap \forall r.B$ is subsumed by $A \sqcap \exists r.B$.

10. $A \sqcap \exists r.B$ is subsumed by $A \sqcap \forall r.B$.

Problem 3. *Extend the knowledge base you built in Problem 2 from the previous Exercise Sheet to capture the following statements. You may use additional features such as number restrictions if needed; you may also need more than one axiom for some of the statements.*

1. *Cars have between three and four wheels.*

2. *Bicycles have exactly two wheels.*

3. *A human who legally controls a car holds a driving license and is an adult (this is a difficult one!).*

4. *A vehicle is controlled by exactly one human.*

5. *A thing's parts' parts are that thing's parts.*

6. *A car with a broken part is broken.*

7. *Bob controls a car with a wheel that has a broken axle.*