Exercise Sheet 6: Trakhtenbrot's Theorem

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Exercise 6.1. Use Trakhtenbrot's Theorem to show that the following problems are undecidable by reducing finite satisfiability to each of them:

1. FO query containment
2. FO query emptiness
3. Domain independence of FO queries

Exercise 6.2. In the lecture, we have seen a logical formula that is finitely satisfiable if and only if the given deterministic Turing machine (DTM) halts after finitely many steps on the given input.

For each of the following statements, decide if it is true or false.

1. If the formula has a model at all, then this model is finite.
2. Every model contains a "start configuration": a right-sequence of elements ("cells") that are not reachable from any other cell via future, and where there is a first element in the chain (a cell with no element to its left).
3. Every model contains exactly one such start configuration.
4. If a cell is reachable from the first cell of the start configuration via future, then it does not have a cell on its left.
5. The future of a cell's neighbour is equal to the neighbour of the cell's future.
6. If the Turing machine halts on the input, then every model of the formula is finite.
7. For every model of the formula holds that no element is in a future relation with itself.

Exercise 6.3. Apply the conjunctive query minimisation algorithm to find a core of the following CQs:

1. $\exists x, y, z.\ R(x, y) \wedge R(x, z)$
2. $\exists x, y, z.\ R(x, y) \wedge R(x, z) \wedge R(y, z)$
3. $\exists x, y, z.\ R(x, y) \wedge R(x, z) \wedge R(y, z) \wedge R(x, x)$
4. $\exists v, w.\ S(x, a, y) \wedge S(x, v, y) \wedge S(x, w, y) \wedge S(x, x, x)$

Exercise 6.4. Explain why the CQ minimisation algorithm is correct:

1. Why is the result guaranteed to be a minimal CQ?
2. Why is the result guaranteed to be unique up to bijective renaming of variables?