Answer Set Programming: Solving

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Motivation

Outline

1 Motivation

- 2 Boolean constraints
- 3 Nogoods from logic programs
 Nogoods from program completion
 Nogoods from loop formulas
- 4 Conflict-driven nogood learning
 - CDNL-ASP Algorithm
 - Nogood Propagation
 - Conflict Analysis

Motivation

- Goal Approach to computing stable models of logic programs, based on concepts from
 - Constraint Processing (CP) and
 - Satisfiability Testing (SAT)
- Idea View inferences in ASP as unit propagation on nogoods

Benefits

- A uniform constraint-based framework for different kinds of inferences in ASP
- Advanced techniques from the areas of CP and SAT
- Highly competitive implementation

Boolean constraints

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An assignment A over $dom(A) = atom(P) \cup body(P)$ is a sequence

 $(\sigma_1,\ldots,\sigma_n)$

of signed literals σ_i of form $\mathsf{T}v$ or $\mathsf{F}v$ for $v \in dom(A)$ and $1 \le i \le n$ **T**v expresses that v is *true* and $\mathsf{F}v$ that it is *false*

The complement, $\overline{\sigma}$, of a literal σ is defined as $\overline{\mathsf{T}v} = \mathsf{F}v$ and $\overline{\mathsf{F}v} = \mathsf{T}v$ A $\circ \sigma$ stands for the result of appending σ to A

Given $A = (\sigma_1, \ldots, \sigma_{k-1}, \sigma_k, \ldots, \sigma_n)$, we let $A[\sigma_k] = (\sigma_1, \ldots, \sigma_{k-1})$

We sometimes identify an assignment with the set of its literals

Given this, we access *true* and *false* propositions in A via

 $A^{\mathsf{T}} = \{v \in dom(A) \mid \mathsf{T}v \in A\}$ and $A^{\mathsf{F}} = \{v \in dom(A) \mid \mathsf{F}v \in A\}$

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- A nogood is a set {σ₁,..., σ_n} of signed literals, expressing a constraint violated by any assignment containing σ₁,..., σ_n
- An assignment A such that $A^{\mathsf{T}} \cup A^{\mathsf{F}} = dom(A)$ and $A^{\mathsf{T}} \cap A^{\mathsf{F}} = \emptyset$ is a solution for a set Δ of nogoods, if $\delta \not\subseteq A$ for all $\delta \in \Delta$
- For a nogood δ , a literal $\sigma \in \delta$, and an assignment A, we say that $\overline{\sigma}$ is unit-resulting for δ wrt A, if

1 $\delta \setminus A = \{\sigma\}$ and 2 $\overline{\sigma} \notin A$

For a set Δ of nogoods and an assignment A, unit propagation is the iterated process of extending A with unit-resulting literals until no further literal is unit-resulting for any nogood in Δ

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Nogoods from logic programs

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The completion of a logic program P can be defined as follows:

$$\{ v_B \leftrightarrow a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n \mid \\ B \in body(P) \text{ and } B = \{ a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n \} \} \\ \cup \ \{ a \leftrightarrow v_{B_1} \vee \cdots \vee v_{B_k} \mid \\ a \in atom(P) \text{ and } body_P(a) = \{ B_1, \dots, B_k \} \} ,$$

where $body_P(a) = \{body(r) \mid r \in P \text{ and } head(r) = a\}$

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The (body-oriented) equivalence

 $v_B \leftrightarrow a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n$

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can be decomposed into two implications:

 $1 \quad v_B \to a_1 \land \cdots \land a_m \land \neg a_{m+1} \land \cdots \land \neg a_n$

is equivalent to the conjunction of

 $\neg v_B \lor a_1, \ldots, \neg v_B \lor a_m, \neg v_B \lor \neg a_{m+1}, \ldots, \neg v_B \lor \neg a_n$

and induces the set of nogoods

 $\Delta(B) = \{ \{ \mathsf{T}B, \mathsf{F}a_1 \}, \dots, \{ \mathsf{T}B, \mathsf{F}a_m \}, \{ \mathsf{T}B, \mathsf{T}a_{m+1} \}, \dots, \{ \mathsf{T}B, \mathsf{T}a_n \} \}$

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The (body-oriented) equivalence

 $v_B \leftrightarrow a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n$

can be decomposed into two implications:

2 $a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n \rightarrow v_B$

gives rise to the nogood

 $\delta(B) = \{\mathsf{F}B, \mathsf{T}a_1, \ldots, \mathsf{T}a_m, \mathsf{F}a_{m+1}, \ldots, \mathsf{F}a_n\}$

Analogously, the (atom-oriented) equivalence

$$a \leftrightarrow v_{B_1} \vee \cdots \vee v_{B_k}$$

yields the nogoods

1
$$\Delta(a) = \{ \{ Fa, TB_1 \}, \dots, \{ Fa, TB_k \} \}$$
 and

$$2 \ \delta(a) = \{\mathsf{T}a, \mathsf{F}B_1, \dots, \mathsf{F}B_k\}$$

• For an atom a where $body_P(a) = \{B_1, \ldots, B_k\}$, we get

 $\{\mathsf{T}a, \mathsf{F}B_1, \dots, \mathsf{F}B_k\}$ and $\{\{\mathsf{F}a, \mathsf{T}B_1\}, \dots, \{\mathsf{F}a, \mathsf{T}B_k\}\}$

• Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

 $\begin{array}{rcl} x & \leftarrow & y \\ x & \leftarrow & \sim z \end{array} & \left\{ \mathsf{T}x, \mathsf{F}\{y\}, \mathsf{F}\{\sim z\} \right\} \\ \left\{ \{\mathsf{F}x, \mathsf{T}\{y\}\}, \{\mathsf{F}x, \mathsf{T}\{\sim z\}\} \right\} \end{array}$

For nogood $\{Tx, F\{y\}, F\{\sim z\}\}$, the signed literal

For an atom a where $body_P(a) = \{B_1, \dots, B_k\}$, we get

 $\{Ta, FB_1, \dots, FB_k\}$ and $\{\{Fa, TB_1\}, \dots, \{Fa, TB_k\}\}$

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For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

Fx is unit-resulting wrt assignment (F{y}, F{~z}) and
 T{~z} is unit-resulting wrt assignment (Tx, F{y})

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• Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

$x \leftarrow y$	$\{Tx,F\{y\},F\{\sim z\}\}$
$x \leftarrow \sim z$	$\{\{Fx,T\{y\}\},\{Fx,T\{\sim z\}\}\}$

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For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- F_x is unit-resulting wrt assignment $(F\{y\}, F\{\sim z\})$ and
- **T** $\{\sim z\}$ is unit-resulting wrt assignment $(Tx, F\{y\})$

For an atom *a* where $body_P(a) = \{B_1, \dots, B_k\}$, we get

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• Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

$$\begin{array}{l} x \leftarrow y \\ x \leftarrow -z \end{array} \qquad \{ \mathsf{T}x, \mathsf{F}\{y\}, \mathsf{F}\{\sim z\} \} \\ \{ \mathsf{F}x, \mathsf{T}\{y\}, \{\mathsf{F}x, \mathsf{T}\{\sim z\} \} \} \end{array}$$

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For nogood $\{Tx, F\{y\}, F\{\sim z\}\}$, the signed literal **F**x is unit-resulting wrt assignment $(F\{y\}, F\{\sim z\})$ and **T** $\{\sim z\}$ is unit-resulting wrt assignment $(Tx, F\{y\})$

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For nogood {Tx, F{y}, F{~z}}, the signed literal
Fx is unit-resulting wrt assignment (F{y}, F{~z}) and
T{~z} is unit-resulting wrt assignment (Tx, F{y})

Nogoods from logic programs body-oriented nogoods

• For a body $B = \{a_1, \ldots, a_m, \sim a_{m+1}, \ldots, \sim a_n\}$, we get

$$\{FB, Ta_1, \dots, Ta_m, Fa_{m+1}, \dots, Fa_n \} \\ \{ \{TB, Fa_1\}, \dots, \{TB, Fa_m\}, \{TB, Ta_{m+1}\}, \dots, \{TB, Ta_n\} \}$$

• Example Given Body $\{x, \sim y\}$, we obtain



{F{ $x, \sim y$ }, Tx, Fy} { {T{ $x, \sim y$ }, Fx}, {T{ $x, \sim y$ }, Ty} }

For nogood $\delta(\{x, \sim y\}) = \{F\{x, \sim y\}, Tx, Fy\}$, the signed literal **T** $\{x, \sim y\}$ is unit-resulting wrt assignment (Tx, Fy) and **T**y is unit-resulting wrt assignment (F $\{x, \sim y\}, Tx$)

Nogoods from logic programs body-oriented nogoods

For a body $B = \{a_1, \ldots, a_m, \sim a_{m+1}, \ldots, \sim a_n\}$, we get

{FB, Ta₁,..., Ta_m, Fa_{m+1},..., Fa_n} { {TB, Fa₁},..., {TB, Fa_m}, {TB, Ta_{m+1}}, ..., {TB, Ta_n} }

Example Given Body $\{x, \sim y\}$, we obtain



$$\{F\{x, \sim y\}, Tx, Fy\} \\ \{ \{T\{x, \sim y\}, Fx\}, \{T\{x, \sim y\}, Ty\} \}$$

For nogood δ({x, ~y}) = {F{x, ~y}, Tx, Fy}, the signed literal
T{x, ~y} is unit-resulting wrt assignment (Tx, Fy) and
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Sebastian Rudolph (TUD)

Nogoods from logic programs body-oriented nogoods

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Example Given Body $\{x, \sim y\}$, we obtain

$$\begin{array}{c} \dots \leftarrow x, \sim y \\ \vdots \\ \dots \leftarrow x, \sim y \end{array} \qquad \{ \mathsf{F}\{x, \sim y\}, \mathsf{T}x, \mathsf{F}y \} \\ \{ \{\mathsf{T}\{x, \sim y\}, \mathsf{F}x\}, \{\mathsf{T}\{x, \sim y\}, \mathsf{T}y\} \} \end{array}$$

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Characterization of stable models for tight logic programs

Let P be a logic program and

$$\begin{array}{lll} \Delta_P & = & \{\delta(a) \mid a \in atom(P)\} \cup \{\delta \in \Delta(a) \mid a \in atom(P)\} \\ & \cup & \{\delta(B) \mid B \in body(P)\} \cup \{\delta \in \Delta(B) \mid B \in body(P)\} \end{array}$$

Theorem Let P be a tight logic program. Then, $X \subseteq atom(P)$ is a stable model of P iff $X = A^{T} \cap atom(P)$ for a (unique) solution A for Δ_{P}

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Characterization of stable models for tight logic programs, ie. free of positive recursion

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- 2 Boolean constraints
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4 Conflict-driven nogood learning

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Nogoods from logic programs via loop formulas

Let P be a normal logic program and recall that:

• For $L \subseteq atom(P)$, the external supports of L for P are $ES_P(L) = \{r \in P \mid head(r) \in L \text{ and } body(r)^+ \cap L = \emptyset\}$

• The (disjunctive) loop formula of L for P is

$$LF_{P}(L) = (\bigvee_{A \in L} A) \to (\bigvee_{r \in ES_{P}(L)} body(r))$$

$$\equiv (\bigwedge_{r \in ES_{P}(L)} \neg body(r)) \to (\bigwedge_{A \in L} \neg A)$$

Note The loop formula of L enforces all atoms in L to be false whenever L is not externally supported

■ The external bodies of L for P are EB_P(L) = {body(r) | r ∈ ES_P(L)}

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Nogoods from logic programs loop nogoods

■ For a logic program P and some Ø ⊂ U ⊆ atom(P), define the loop nogood of an atom a ∈ U as

$$\lambda(a, U) = \{ \mathsf{T}a, \mathsf{F}B_1, \dots, \mathsf{F}B_k \}$$

where $EB_P(U) = \{ B_1, \dots, B_k \}$

• We get the following set of loop nogoods for *P*:

$$\Lambda_P = \bigcup_{\emptyset \subset U \subseteq atom(P)} \{\lambda(a, U) \mid a \in U\}$$

The set Λ_P of loop nogoods denies cyclic support among true atoms

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For a logic program P and some $\emptyset \subset U \subseteq atom(P)$, define the loop nogood of an atom $a \in U$ as

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Example

Consider the program

$$\left\{\begin{array}{ccc} x \leftarrow \sim y & u \leftarrow x \\ y \leftarrow \sim y & u \leftarrow v \\ y \leftarrow \sim x & v \leftarrow u, y \end{array}\right\}$$

• For u in the set $\{u, v\}$, we obtain the loop nogood: $\lambda(u, \{u, v\}) = \{\mathsf{T}u, \mathsf{F}\{x\}\}$ Similarly for v in $\{u, v\}$, we get: $\lambda(v, \{u, v\}) = \{\mathsf{T}v, \mathsf{F}\{x\}\}$

Example

Consider the program

$$\left\{\begin{array}{ccc} x \leftarrow \neg y & u \leftarrow x \\ x \leftarrow \neg y & u \leftarrow v \\ y \leftarrow \neg x & v \leftarrow u, y \end{array}\right\}$$

For u in the set {u, v}, we obtain the loop nogood:
 λ(u, {u, v}) = {Tu, F{x}}
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Example

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Characterization of stable models

Theorem

Let P be a logic program. Then, $X \subseteq atom(P)$ is a stable model of P iff $X = A^{T} \cap atom(P)$ for a (unique) solution A for $\Delta_{P} \cup \Lambda_{P}$

Some remarks

Nogoods in Λ_P augment Δ_P with conditions checking for unfounded sets, in particular, those being loops
 While |Δ_P| is linear in the size of P, Λ_P may contain exponentially many (non-redundant) loop nogoods

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Conflict-driven nogood learning

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Towards conflict-driven search

Boolean constraint solving algorithms pioneered for SAT led to:

- Traditional DPLL-style approach (DPLL stands for 'Davis-Putnam-Logemann-Loveland')
 - (Unit) propagation
 - (Chronological) backtracking
 - in ASP, eg *smodels*
- Modern CDCL-style approach (CDCL stands for 'Conflict-Driven Constraint Learning')
 - (Unit) propagation
 - Conflict analysis (via resolution)
 - $\blacksquare Learning + Backjumping + Assertion$
 - in ASP, eg *clasp*

DPLL-style solving

loop

propagate

// deterministically assign literals

\boldsymbol{if} no conflict \boldsymbol{then}

if all variables assigned then return solution
else decide // non-deterministically assign some literal

else

 $\boldsymbol{\mathsf{if}}$ top-level conflict $\boldsymbol{\mathsf{then}}\ \boldsymbol{\mathsf{return}}\ \mathsf{unsatisfiable}\ \boldsymbol{\mathsf{else}}$

backtrack // unassign literals propagated after last decision flip // assign complement of last decision literal

CDCL-style solving

loop

propagate

// deterministically assign literals

\boldsymbol{if} no conflict \boldsymbol{then}

if all variables assigned then return solution
else decide // non-deterministically assign some literal

else

 ${\bf if}$ top-level conflict ${\bf then}\ {\bf return}\ {\bf unsatisfiable}\ {\bf else}$

analyze // analyze conflict and add conflict constraint backjump // unassign literals until conflict constraint is unit

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Outline of CDNL-ASP algorithm

Keep track of deterministic consequences by unit propagation on:

- Program completion
- Loop nogoods, determined and recorded on demand
- Dynamic nogoods, derived from conflicts and unfounded sets
- When a nogood in $\Delta_P \cup \nabla$ becomes violated:
 - Analyze the conflict by resolution (until reaching a Unique Implication Point, short: UIP)
 - \blacksquare Learn the derived conflict nogood δ
 - Backjump to the earliest (heuristic) choice such that the complement of the UIP is unit-resulting for δ
 - Assert the complement of the UIP and proceed (by unit propagation)
- Terminate when either:
 - Finding a stable model (a solution for $\Delta_P \cup \Lambda_P$)
 - Deriving a conflict independently of (heuristic) choices

 $\begin{bmatrix} \Delta_P \\ [\Lambda_P] \\ [\nabla] \end{bmatrix}$

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Algorithm 1: CDNL-ASP

: A normal program P Input Output • A stable model of P or "no stable model" $A := \emptyset$ // assignment over $atom(P) \cup body(P)$ $\nabla := \emptyset$ // set of recorded nogoods dl := 0// decision level loop $(A, \nabla) := \text{NOGOODPROPAGATION}(P, \nabla, A)$ if $\varepsilon \subset A$ for some $\varepsilon \in \Delta_P \cup \nabla$ then // conflict if $max(\{dlevel(\sigma) \mid \sigma \in \varepsilon\} \cup \{0\}) = 0$ then return no stable model $(\delta, dI) := \text{CONFLICTANALYSIS}(\varepsilon, P, \nabla, A)$ $\nabla := \nabla \cup \{\delta\}$ // (temporarily) record conflict nogood $A := A \setminus \{ \sigma \in A \mid dl < dlevel(\sigma) \}$ // backjumping else if $A^{\mathsf{T}} \cup A^{\mathsf{F}} = atom(P) \cup body(P)$ then // stable model return $A^{\mathsf{T}} \cap atom(P)$ else $\sigma_d := \text{SELECT}(P, \nabla, A)$ // decision dl := dl + 1 $dlevel(\sigma_d) := dl$ $A := A \circ \sigma_d$

Observations

- Decision level *dl*, initially set to 0, is used to count the number of heuristically chosen literals in assignment *A*
- For a heuristically chosen literal $\sigma_d = \mathbf{T}a$ or $\sigma_d = \mathbf{F}a$, respectively, we require $a \in (atom(P) \cup body(P)) \setminus (A^{\mathbf{T}} \cup A^{\mathbf{F}})$
- For any literal $\sigma \in A$, $dI(\sigma)$ denotes the decision level of σ , viz. the value dI had when σ was assigned
- A conflict is detected from violation of a nogood $arepsilon \subseteq \Delta_P \cup
 abla$
- A conflict at decision level 0 (where A contains no heuristically chosen literals) indicates non-existence of stable models
- A nogood δ derived by conflict analysis is asserting, that is, some literal is unit-resulting for δ at a decision level k < dl
 - After learning δ and backjumping to decision level k, at least one literal is newly derivable by unit propagation
 - No explicit flipping of heuristically chosen literals !

Sebastian Rudolph (TUD)

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Consider

$$P = \left\{ \begin{array}{cccc} x \leftarrow \neg y & u \leftarrow x, y & v \leftarrow x & w \leftarrow \neg x, \neg y \\ y \leftarrow \neg x & u \leftarrow v & v \leftarrow u, y \end{array} \right\}$$

dl	σ_d	$\overline{\sigma}$	δ
1	Tu		
2	$F\{\sim x, \sim y\}$		
		Fw	$\{Tw,F\{\sim x,\sim y\}\}=\delta(w)$
3	F{~y}	Fx F{x} F{x,y}	$\{\mathbf{T}x, \mathbf{F}\{\sim y\}\} = \delta(x)$ $\{\mathbf{T}\{x\}, \mathbf{F}x\} \in \Delta(\{x\})$ $\{\mathbf{T}\{x, y\}, \mathbf{F}x\} \in \Delta(\{x, y\})$ \vdots $\{\mathbf{T}u, \mathbf{F}\{x\}, \mathbf{F}\{x, y\}\} = \lambda(u, \{u, v\})$

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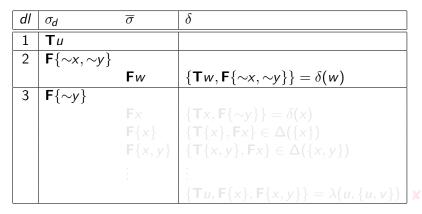
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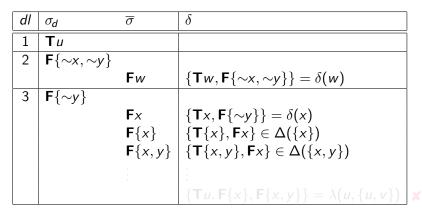
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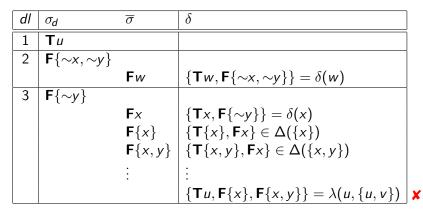
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		Tx	$\{Tu,Fx\}\in abla$
		÷	:
		Tν	$\{Fv,T\{x\}\}\in\Delta(v)$
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		Fw	$\{Tw,F\{\sim x,\sim y\}\}=\delta(w)$

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dl	σ_{d}	$\overline{\sigma}$	δ
1	Тu		
		Tx	$\{Tu,Fx\}\in abla$
		÷	:
		Tν	$\{Fv,T\{x\}\}\in\Delta(v)$
		Fy	$\{Ty,F\{\sim x\}\}=\delta(y)$
		Fw	$\{Tw,F\{\sim x,\sim y\}\}=\delta(w)$

Outline

Motivation

Boolean constraints 2

Nogoods from logic programs 3

- Nogoods from program completion
- Nogoods from loop formulas

4 Conflict-driven nogood learning

- CDNL-ASP Algorithm
- Nogood Propagation
- Conflict Analysis

Derive deterministic consequences via:

- Unit propagation on Δ_P and ∇ ;
- Unfounded sets $U \subseteq atom(P)$
- Note that U is unfounded if $EB_P(U) \subseteq A^F$
 - Note For any $a \in U$, we have $(\lambda(a, U) \setminus \{\mathsf{T}a\}) \subseteq A$

An "interesting" unfounded set U satisfies:

 $\emptyset \subset U \subseteq (atom(P) \setminus A^{\mathsf{F}})$

Wrt a fixpoint of unit propagation, such an unfounded set contains some loop of P
 Note Tight programs do not yield "interesting" unfounded sets !
 Given an unfounded set U and some a ∈ U, adding λ(a, U) to ∇ triggers a conflict or further derivations by unit propagation
 Note Add loop nogoods atom by atom to eventually falsify all a ∈

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Algorithm 2: NOGOODPROPAGATION

Input : A normal program P, a set ∇ of nogoods, and an assignment A.

Output : An extended assignment and set of nogoods.

$$U := \emptyset$$
 // unfounded set

loop

$$\begin{array}{l} \text{repeat} \\ & \text{if } \delta \subseteq A \text{ for some } \delta \in \Delta_P \cup \nabla \text{ then return } (A, \nabla) \qquad // \text{ conflict} \\ & \Sigma := \{\delta \in \Delta_P \cup \nabla \mid \delta \setminus A = \{\overline{\sigma}\}, \sigma \notin A\} \qquad // \text{ unit-resulting nogoods} \\ & \text{if } \Sigma \neq \emptyset \text{ then let } \overline{\sigma} \in \delta \setminus A \text{ for some } \delta \in \Sigma \text{ in} \\ & \left\lfloor \begin{array}{c} dlevel(\sigma) := \max(\{dlevel(\rho) \mid \rho \in \delta \setminus \{\overline{\sigma}\}\} \cup \{0\}) \\ & \bot A := A \circ \sigma \end{array} \right. \\ & \text{until } \Sigma = \emptyset \\ & \text{if } loop(P) = \emptyset \text{ then return } (A, \nabla) \\ & U := U \setminus A^{\mathsf{F}} \\ & \text{if } U = \emptyset \text{ then return } (A, \nabla) \\ & \text{if } U = \emptyset \text{ then return } (A, \nabla) \qquad // \text{ no unfounded set } \emptyset \subset U \subseteq atom(P) \setminus A^{\mathsf{F}} \\ & \text{let } a \in U \text{ in} \\ & \left\lfloor \begin{array}{c} \nabla := \nabla \cup \{\{\mathsf{T}a\} \cup \{\mathsf{F}B \mid B \in EB_P(U)\}\} \end{array} \right. \end{pmatrix} \end{array}$$

Requirements for $\operatorname{UnfoundedSet}$

- Implementations of UNFOUNDEDSET must guarantee the following for a result U
 - $U \subseteq (atom(P) \setminus A^{\mathsf{F}})$
 - $EB_P(U) \subseteq A^{\mathbf{F}}$
 - 3 $U = \emptyset$ iff there is no nonempty unfounded subset of $(atom(P) \setminus A^{\mathsf{F}})$
- Beyond that, there are various alternatives, such as:
 - Calculating the greatest unfounded set
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Example: NogoodPropagation

Consider

$$P = \left\{ \begin{array}{cccc} x \leftarrow \neg y & u \leftarrow x, y & v \leftarrow x & w \leftarrow \neg x, \neg y \\ y \leftarrow \neg x & u \leftarrow v & v \leftarrow u, y \end{array} \right\}$$

dl	σ _d	$\overline{\sigma}$	δ]
1	Tu]
2	$\mathbf{F}\{\sim x, \sim y\}$			1
		Fw	$\{Tw,F\{\sim x,\sim y\}\}=\delta(w)$	
3	F {∼ <i>y</i> }			1
		F <i>x</i>	$\{Tx,F\{\sim y\}\}=\delta(x)$	
		$\mathbf{F}\{x\}$	$\{T\{x\},Fx\}\in\Delta(\{x\})$	
		$\mathbf{F}\{x, y\}$	$\{T\{x,y\},Fx\}\in\Delta(\{x,y\})$	
		T {∼x}	$\{\mathbf{F}\{\sim x\},\mathbf{F}x\}=\delta(\{\sim x\})$	
		Тy	$\{F\{\sim y\},Fy\}=\delta(\{\sim y\})$	
		$T\{v\}$	$\{Tu,F\{x,y\},F\{v\}\}=\delta(u)$	
		$T{u, y}$	$\{\mathbf{F}\{u, y\}, \mathbf{T}u, \mathbf{T}y\} = \delta(\{u, y\})$	
		Tν	$\{Fv,T\{u,y\}\}\in\Delta(v)$	
			$\{T u, F \{x\}, F \{x, y\}\} = \lambda(u, \{u, v\})$	×

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Outline of ConflictAnalysis

- Conflict analysis is triggered whenever some nogood δ ∈ Δ_P ∪ ∇ becomes violated, viz. δ ⊆ A, at a decision level dl > 0
 - Note that all but the first literal assigned at *dl* have been unit-resulting for nogoods ε ∈ Δ_P ∪ ∇
 - If σ ∈ δ has been unit-resulting for ε, we obtain a new violated nogood by resolving δ and ε as follows:

$(\delta \setminus \{\sigma\}) \cup (\varepsilon \setminus \{\overline{\sigma}\})$

Resolution is directed by resolving first over the literal $\sigma \in \delta$ derived last, viz. $(\delta \setminus A[\sigma]) = \{\sigma\}$

Iterated resolution progresses in inverse order of assignment

Iterated resolution stops as soon as it generates a nogood δ containing exactly one literal σ assigned at decision level dl

This literal σ is called First Unique Implication Point (First-UIP)

All literals in $(\delta \setminus \{\sigma\})$ are assigned at decision levels smaller than dl

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Algorithm 3: CONFLICTANALYSIS

Input : A non-empty violated nogood δ , a normal program *P*, a set ∇ of nogoods, and an assignment *A*.

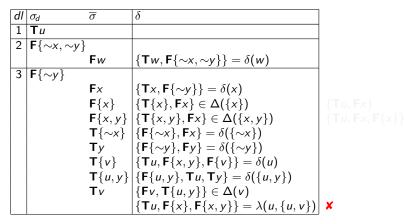
Output : A derived nogood and a decision level.

loop

$$\begin{array}{|c|c|c|c|c|} \mbox{let } \sigma \in \delta \mbox{ such that } \delta \setminus A[\sigma] = \{\sigma\} \mbox{ in } \\ k := \max(\{dlevel(\rho) \mid \rho \in \delta \setminus \{\sigma\}\} \cup \{0\}) \\ \mbox{if } k = dlevel(\sigma) \mbox{ then } \\ \mbox{let } \varepsilon \in \Delta_P \cup \nabla \mbox{ such that } \varepsilon \setminus A[\sigma] = \{\overline{\sigma}\} \mbox{ in } \\ \mbox{let } \delta := (\delta \setminus \{\sigma\}) \cup (\varepsilon \setminus \{\overline{\sigma}\}) \mbox{ // resolution } \\ \mbox{else return } (\delta, k) \end{array}$$

Consider

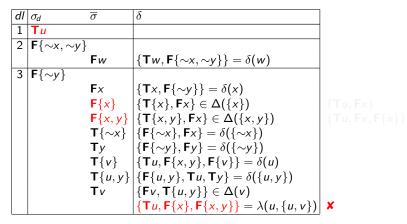
$$P = \left\{ \begin{array}{cccc} x \leftarrow \sim y & u \leftarrow x, y & v \leftarrow x & w \leftarrow \sim x, \sim y \\ y \leftarrow \sim x & u \leftarrow v & v \leftarrow u, y \end{array} \right\}$$



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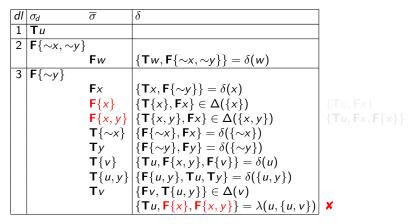
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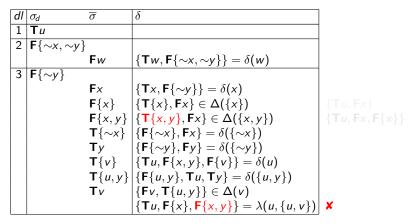
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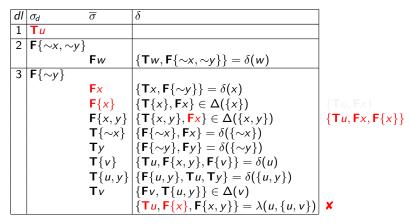
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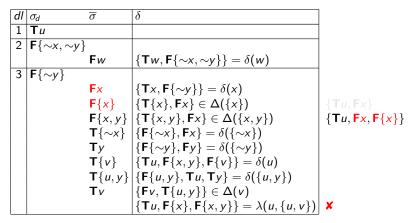
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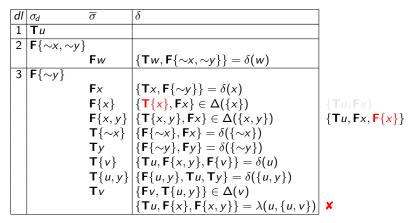
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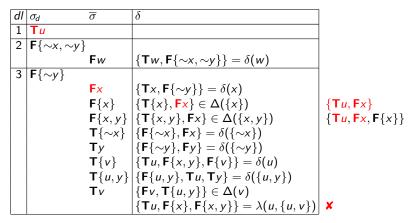
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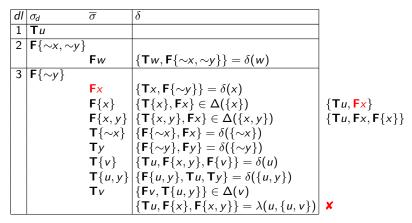
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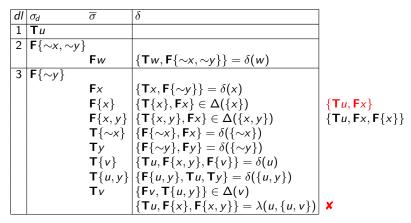
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There always is a First-UIP at which conflict analysis terminates

- In the worst, resolution stops at the heuristically chosen literal assigned at decision level *dI*
- The nogood δ containing First-UIP σ is violated by A, viz. δ ⊆ A
 We have k = max({dl(ρ) | ρ ∈ δ \ {σ}} ∪ {0}) < dl
 - After recording δ in ∇ and backjumping to decision level k, $\overline{\sigma}$ is unit-resulting for δ !
 - Such a nogood δ is called asserting

Asserting nogoods direct conflict-driven search into a different region of the search space than traversed before, without explicitly flipping any heuristically chosen literal !

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