TU Dresden, Fakultät Informatik
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Complexity Theory

Exercise 11: Randomized Computation 2

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Definition. A PTM $\mathcal{M}$ has expected runtime $f : \mathbb{N} \to \mathbb{R}$ if, for any input w, the expectation $E[T_w]$ of the number T_w of steps taken by $\mathcal{M}$ on input w is $T_w \leq f(|w|)$.

Definition. ZPP is the class of all languages for which there is a PTM $\mathcal{M}$ that

- returns the correct answer whenever it halts,
- has expected runtime f for some polynomial function f.

Exercise 11.1. Consider the following alternative definition of ZPP:
"A language **L** is in ZPP iff there exists some polynomial time PTM $\mathcal{M}$ that answers Accept (A), Reject (R), or Inconclusive (I), and:

- For all $w \in \mathbf{L}$, the $\mathcal{M}$ always returns A or I.
- For all $w \notin \mathbf{L}$, the $\mathcal{M}$ always returns R or I.
- For all $w \in \Sigma^*$, *Pr*$[\mathcal{M}(w) = \mathsf{I}] < \frac{1}{2}$."

Show that this definition is equivalent to the definition above.

Exercise 11.2. Prove that $\mathrm{NP} \subseteq \mathrm{PP}$.

Exercise 11.3. Prove Theorem 23.7 (see slide 18 of lecture 23).

Definition. A language **L** is in RL if there is an $O(\log n)$-space PTM $\mathcal{M}$ such that:

- For all $w \in \mathbf{L}$, then *Pr*$[\mathcal{M}(w) = 1] \geq \frac{2}{3}$.
- For all $w \notin \mathbf{L}$, then *Pr*$[\mathcal{M}(w) = 1] = 0$.

Exercise 11.4. Let **UPath** be the set of all tuples $\langle G, s, t\rangle$ with G an undirected, and s and t are two connected vertices in G. Show that **UPath** $\in \mathrm{RL}$.