



# PROBLEM SOLVING AND SEARCH IN ARTIFICIAL INTELLIGENCE

## Lecture 2 Uninformed Search vs. Informed Search

Sarah Gaggl

Dresden, 28th April 2015

# Agenda

- 1 Introduction
- 2 Uninformed Search versus Informed Search (Best First Search, A\* Search, Heuristics)
- 3 Local Search, Stochastic Hill Climbing, Simulated Annealing
- 4 Tabu Search
- 5 Answer-set Programming (ASP)
- 6 Constraint Satisfaction (CSP)
- 7 Structural Decomposition Techniques (Tree/Hypertree Decompositions)
- 8 Evolutionary Algorithms/ Genetic Algorithms

# Traditional Methods

- There are many classic algorithms to search spaces for an optimal solution.
- Broadly, they fall into two disjoint classes:
  - Algorithms that only evaluate **complete solutions** (exhaustive search, local search, ...).
  - Algorithms that require the evaluation of **partially constructed** or approximate solutions.
- Algorithms that treat **complete** solutions can be **stopped any time**, and give at least one **potential answer**.
- If you interrupt an algorithm that works on **partial** solutions, the **results might be useless**.

# Complete Solutions

- All decision variables are specified.
- For example, binary strings of length  $n$  constitute complete solutions for any  $n$ -variable SAT.
- Permutations of  $n$  cities constitute complete solutions for a TSP.
- We can compare two complete solutions using an evaluation function.
- Many algorithms rely on such comparisons, manipulating one single complete solution at a time.
- When a new solution has a better evaluation than the previous best solution, it replaces that prior solution.
- Exhaustive search, local search, hill climbing as well as modern heuristic methods such as simulated annealing, tabu search and evolutionary algorithms fall into this category.

# Partial Solutions

There are two forms:

- 1 **incomplete solution** to the problem originally posed, and
  - 2 **complete solution** to a **reduced** (i.e. simpler) problem.
- **Incomplete solutions** reside in a **subset** of the original problem's **search space**.
    - In an SAT, consider all of the binary strings where the first two variables were assigned the value 1 (i.e. TRUE).
    - In a TSP, consider every permutation of cities that contains the sequence 7 – 11 – 2 – 16.
    - We **fix** the attention on a **subset** of the search space that has a **partial property**.
    - **Hopefully**, that property is also **shared by the real solution!**

# Partial Solutions ctd.

- **Decompose** original problem into a set of **smaller** and **simpler** problems.
  - Hope: solving each of the easier problems and **combine the partial solutions**, results in an answer for the original problem.
  - In a TSP, consider only  $k$  out of  $n$  cities and try to establish the shortest path from city  $i$  to  $j$  that passes through all  $k$  of these cities.
  - **Reduce** the **size of the search space** significantly and search for a complete solution within the restricted domain.
  - Such partial solutions can serve as **building blocks** for the solution to the original problem.

# Partial Solutions ctd.

- **Decompose** original problem into a set of **smaller** and **simpler** problems.
  - Hope: solving each of the easier problems and **combine the partial solutions**, results in an answer for the original problem.
  - In a TSP, consider only  $k$  out of  $n$  cities and try to establish the shortest path from city  $i$  to  $j$  that passes through all  $k$  of these cities.
  - **Reduce** the **size of the search space** significantly and search for a complete solution within the restricted domain.
  - Such partial solutions can serve as **building blocks** for the solution to the original problem.
- But, algorithms that work on partial solutions pose **additional difficulties**. One needs to
  - devise a way to **organize the sub-spaces** so that they can be searched efficiently, and
  - create a **new evaluation function** that can assess the quality of partial solutions.

# Exhaustive Search

- Checks **every** solution in the search space until the **best global** solution has been found.
- Can be used **only for small instances** of problems.
- Exhaustive (**enumerative**) algorithms are **simple**.
- Search space can be reduced by **backtracking**.
- Some optimization methods, e.g., **branch and bound** and **A\*** are based on an exhaustive search.



# Exhaustive Search

- Checks **every** solution in the search space until the **best global** solution has been found.
- Can be used **only for small instances** of problems.
- Exhaustive (**enumerative**) algorithms are **simple**.
- Search space can be reduced by **backtracking**.
- Some optimization methods, e.g., **branch and bound** and **A\*** are based on an exhaustive search.
- **How** can we generate a **sequence** of every possible solution to the problem?
  - The **order** in which the solutions are generated and evaluated is **irrelevant** (because we evaluate all of them).
  - The **answer** for the question depends on the selected **representation**.

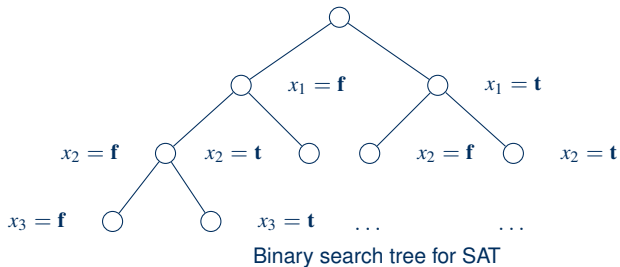
# Enumerating the SAT

- We have to generate **every possible binary string of length  $n$** .
- All solutions **correspond to whole numbers** in a one-to-one mapping.
- Generate all non-negative **integers** from 0 to  $2^n - 1$  and convert each of these integers into the **matching binary string** of length  $n$ .

0000	0	0100	4	1000	8	1100	12
0001	1	0101	5	1001	9	1101	13
0010	2	0110	6	1010	10	1110	14
0011	3	0111	7	1011	11	1111	15

- **Bits** of the string are the **truth assignments** of the decision variables.
- **Organize** the search space, for example **partition** into two disjoint sub-spaces. First contains all the vectors where  $x_1 = \mathbf{f}$  (FALSE), and the second contains all vectors where  $x_1 = \mathbf{t}$  (TRUE).

# Enumerating the SAT ctd.



# Search Strategies

A strategy is defined by picking the **order of node expansion**.

Strategies are evaluated along the following dimensions:

- **Completeness** - does it always find a solution if one exists?
- **Time complexity** - number of nodes generated/expanded.
- **Space complexity** - maximum number of nodes in memory.
- **Optimality** - does it always find a least-cost solution?

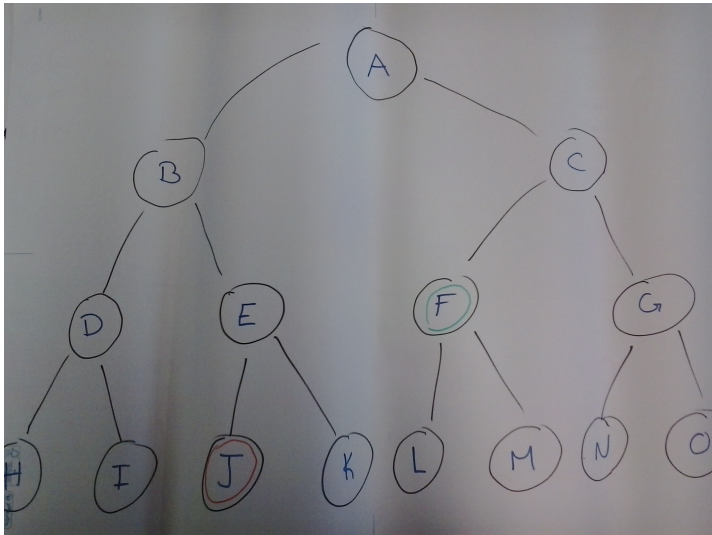
Time and space complexity are measured in terms of

- $b$  - maximum branching factor of the search tree;
- $d$  - depth of the least-cost solution;
- $m$  - maximum depth of the state space (may be  $\infty$ ).

# Uninformed Search

	Breadth-First-Search	Depth-First-Search	Iterative Deepening
	node expansion by level * FIFO	node expansion by first child * LIFO	iterative increasing of depth-limited search
Complete	Yes	No	Yes
TIME	$O(b^d)$	$O(b^m)$	$O(b^d)$
Space	$O(b^d)$	$O(bm)$	$O(bd)$
Optimal	Yes	No	Yes

# Uninformed Search ctd.



# Informed Search

Greedy

$f(n) = h(n)$

- incomplete for infinite search spaces

Informed Search

- uses information beyond the problem definition (usually heuristic function)
- best first search: expands node with minimal  $f(n)$

Heuristic  $h(n)$  estimates cost from current node

- admissible
- consistent

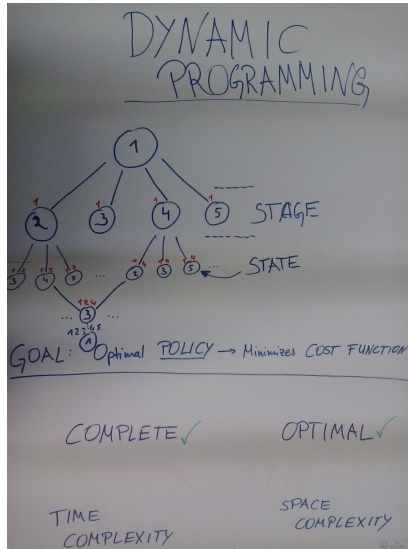
$A^*$

$f(n) = g(n) + h(n)$

↑  
cost to current node

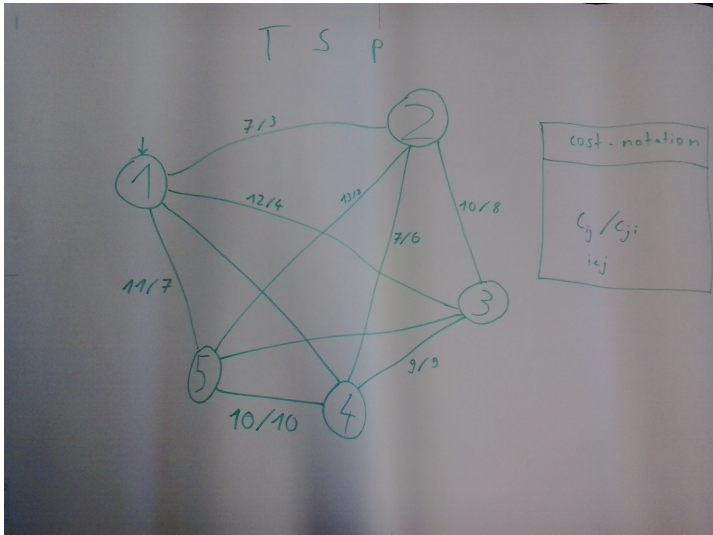
- optimal
- complete

# Dynamic Programming





# Dynamic Programming



# References



Zbigniew Michalewicz and David B. Fogel.

**How to Solve It: Modern Heuristics**, volume 2. Springer, 2004.



Stuart J. Russell and Peter Norvig.

**Artificial Intelligence - A Modern Approach (3. edition)**. Pearson Education, 2010.