Exercise 10.1. Denote with \( \text{add} \): \( \{ 0, 1 \}^{2n} \rightarrow \{ 0, 1 \}^{n+1} \) the function that takes two binary \( n \)-bit numbers \( x \) and \( y \) and returns their \( n + 1 \)-bit sum. Show that \( \text{add} \) can be computed with size \( O(n) \) circuits.

Exercise 10.2. Define the function \( \text{maj}_n : \{ 0, 1 \}^n \rightarrow \{ 0, 1 \}^n \) by
\[
\text{maj}_n(x_1, \ldots, x_n) := \begin{cases} 
0 & \text{if } \sum x_i < n/2 \\
1 & \text{if } \sum x_i \geq n/2.
\end{cases}
\]
Devised a circuit to compute \( \text{maj}_3 \) and test it on the example input 101 and 010.

* Exercise 10.3. Show how to compute \( \text{maj}_n \) with circuits of size \( O(n \log n) \).

Exercise 10.4. Show \( \text{NC}^1 \subseteq L \).

Exercise 10.5. Show that every Boolean function with \( n \) variables can be computed with a circuit of size \( O(n \cdot 2^n) \).

Exercise 10.6. Show that every language \( L \subseteq \{ 1^n \mid n \in \mathbb{N} \} \) is contained in \( P/\text{poly} \). Conclude that \( P/\text{poly} \) contains undecidable languages.

Exercise 10.7. Find a decidable language in \( P/\text{poly} \) that is not contained in \( P \).

Hint:
\( \text{EXPTIME} \) is contained in \( 5\text{EXP} \) if \( \text{EXP} \subseteq \text{P/poly} \) and \( \text{EXP} \) is EXP-complete under polynomial-time reductions.