Exercise Sheet 7: FO Query Expressivity

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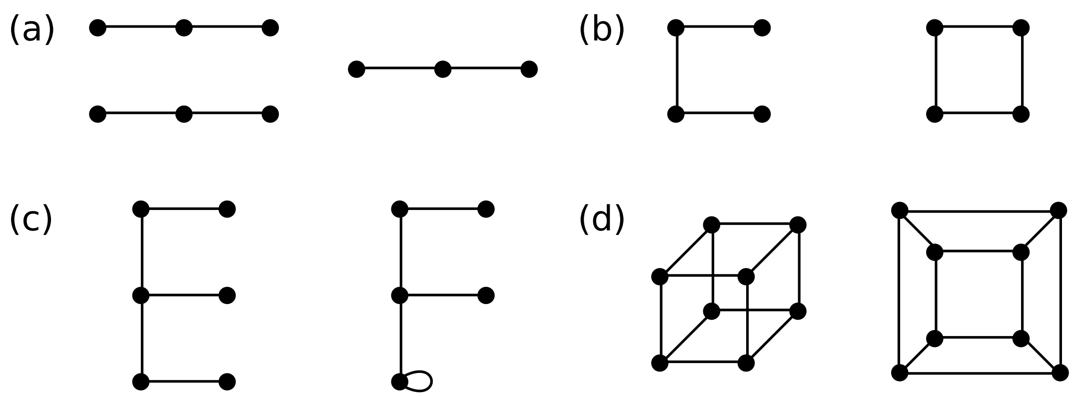
Database Theory, 2022-05-24, Summer Term 2022

Exercise 7.1. For the following pairs of structures, find the maximal r such that $\mathcal{I} \sim_r \mathcal{J}$:

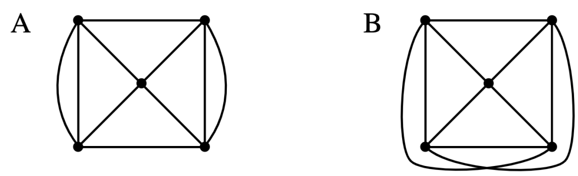


Exercise 7.2. A *linear order* is a relational structure with one binary relational symbol $\leq$ that is interpreted as a reflexive, asymmetric, transitive and total relation over the domain. Up to renaming of domain elements there is exactly one linear order for every finite domain, which can be depicted as a chain of elements. We denote the linear order of size n by $\mathcal{L}_n$. For example:

$$\mathcal{L}_6 : 1 \leq 2 \leq 3 \leq 4 \leq 5 \leq 6 \qquad \text{and} \qquad \mathcal{L}_7 : 1 \leq 2 \leq 3 \leq 4 \leq 5 \leq 6 \leq 7$$

1. For which r are $\mathcal{L}_6 \sim_r \mathcal{L}_7$?
2. More generally, for which r are $\mathcal{L}_n \sim_r \mathcal{L}_{n+1}$? $(*)$

Exercise 7.3. A graph is *planar* if it can be drawn on the plane without intersections of edges. For example, the following graph A is planar, while graph B is not:



Can the graphs A and B be distinguished by a first-order query? Show that planarity is not FO-definable by using locality.