Incorporating Stage Semantics in the SCC-recursive Schema for Argumentation Semantics*

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Abstract

Recently, the stage and *cf2* semantics for abstract argumentation attracted specific attention. By distancing from the notion of defense, they are capable to select arguments out of odd-length cycles. Furthermore, the maximality criterion of naive sets ensures reasonable solutions. The SCC-recursive schema, where the *cf2* semantics is defined in, guarantees that some specific evaluation criteria, like directionality, weak- and $\mathcal{CF}$-reinstatement, are fulfilled. Beside several desirable properties, both stage and *cf2* semantics still have some drawbacks. The stage semantics does not satisfy the above mentioned evaluation criteria, whereas *cf2* semantics produces some questionable results on frameworks with cycles of length ≥ 6. That's why we suggest to combine stage semantics with the SCC-recursive schema of *cf2* semantics. The resulting *stage2* semantics overcomes the problems regarding *cf2* semantics and still fulfills the mentioned evaluation criteria. Furthermore, we analyze redundant patterns for *stage2* semantics, and we provide a complexity analysis of the associated reasoning problems.

Introduction

For abstract argumentation frameworks (AFs) there are many semantics with different requirements, properties and meanings. In this work we concentrate on semantics based on maximal conflict-free sets, so called naive sets.

While traditional argumentation semantics build on the concept of admissible sets, i.e. sets where each argument attacking an argument in the set is also attacked by the set, recently, some semantics raised attention because they abandon the notion of admissibility and build on naive sets. Following this observation, a distinction is drawn between these kinds of semantics, the so called *admissible-based* and *naive-based* semantics.

Recent investigations (Baroni, Giacomin, and Guida 2005; Baroni and Giacomin 2007; Bodanza and Tohm 2009; Bench-Capon 2003) showed, in certain situations the admissible-based semantics do not provide satisfying results. For instance the appearance of odd-length cycles and in particular self-attacking arguments as a special case of them, have a strong and sometimes undesired influence on the computation of solutions. None of the admissible-based semantics is able to select arguments of such a cycle as accepted, and moreover, they reject arguments just because they are attacked by a self-attacking argument. The reason for this behavior is that in an odd-length cycle, arguments defend their own attacker. As naive-based semantics do not rely on the notion of defense, one can accept both, arguments in an odd-length cycle, as well as arguments attacked by such arguments. One attempt to treat odd- and even-length cycles in a uniform way is met by *cf2* semantics, built on the SCC-recursive schema of Baroni et al. (Baroni, Giacomin, and Guida 2005). However, *cf2* semantics deal odd-length cycles in a more sensitive way, the evaluation of odd-cycle-free AFs e.g. if even-length cycles occur, is now questionable (Gabbay 2012; Gaggl and Woltran 2012). On the other side, stage semantics (Verheij 1996) can also handle odd-length cycles and does not change the behavior of odd-cycle-free AFs. The disadvantages of stage semantics are that very basic properties are not satisfied, for example the skeptical acceptance of unattacked arguments, i.e. the weak reinstatement property (Baroni and Giacomin 2007) is violated.

While naive-based semantics seem to be the right candidates when the above described behavior of admissible-based semantics is unwanted, there are several shortcomings with existing approaches, as mentioned above. To overcome those problems we propose a new semantics combining concepts from *cf2* and stage semantics, which we name *stage2*.

The contributions of this work are the following:

- We have a closer look at the properties of, and differences between, existing naive-based semantics stage and *cf2*, highlighting their shortcomings.
- We suggest to combine the concepts of stage and *cf2* semantics, where we use the SCC-recursive schema of *cf2* semantics and instantiate the base case with stage semantics. In this way, we obtain the novel *stage2* semantics.
- We point out the basic properties of the novel semantics and show that it solves most of the above mentioned problems. In particular, we evaluate *stage2* semantics with the criteria proposed by Baroni and Giacomin (2007).
- Moreover, we analyze redundant patterns w.r.t. *stage2* semantics, where it turns out that it is the second semantics,

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beside *cf2*, satisfying the succinctness property (Gaggl and Woltran 2012).

- Finally, we study computational properties of *stage2* semantics, providing a complexity analysis for the standard argumentation reasoning tasks. We show that complexity from stage semantics carries over to *stage2* semantics and therefore *stage2* is among the computationally hardest argumentation semantics.

Preliminaries

In this section we introduce the basics of abstract argumentation, the semantics we need for further investigations followed by a comparison of *cf2* and stage semantics.

Abstract Argumentation

We first give the formal definition of abstract argumentation frameworks as introduced by Dung (1995).

Definition 1. *An* argumentation framework (AF) *is a pair $F = (A, R)$, where A is a finite set of arguments and $R \subseteq A \times A$ is the attack relation. The pair $(a, b) \in R$ means that a attacks b. A set $S \subseteq A$ of arguments attacks b (in F), if there is an $a \in S$, such that $(a, b) \in R$. An argument $a \in A$ is* defended *by $S \subseteq A$ (in F) iff, for each $b \in A$, it holds that, if $(b, a) \in R$, then S attacks b (in F). Moreover, given an AF F, we use A_F to denote the set of it is arguments and resp. R_F to denote its attack relation.*

The inherent conflicts between the arguments are solved by selecting subsets of arguments, where a semantics σ assigns a collection of sets of arguments to an AF F. The basic requirement for all semantics is that none of the selected arguments attack each other.

Definition 2. *Let $F = (A, R)$ be an AF. A set $S \subseteq A$ is said to be* conflict-free *(in F), if there are no $a, b \in S$, such that $(a, b) \in R$. We denote the collection of sets which are conflict-free (in F) by $cf(F)$. A set $S \subseteq A$ is maximal conflict-free or* naive*, if $S \in cf(F)$ and for each $T \in cf(F)$, $S \not\subset T$. We denote the collection of all naive sets of F by $naive(F)$. For the empty AF $F_0 = (\emptyset, \emptyset)$, we set $naive(F_0) = \{\emptyset\}$.*

Towards the definition of the semantics we introduce the following formal concepts.

Definition 3. *Given an AF $F = (A, R)$ and let $S \subseteq A$. The characteristic function $\mathcal{F}_F : 2^A \to 2^A$ of F is defined as $\mathcal{F}_F(S) = \{x \in A \mid x \text{ is defended by } S\}$. We define the* range *of S as $S_R^+ = S \cup \{b \mid \exists a \in S, \text{ s. t. } (a, b) \in R\}$.*

In the following we give brief definitions of the standard semantics in abstract argumentation (Dung 1995) together with the definition of stage semantics (Verheij 1996). For comprehensive surveys on argumentation semantics the interested reader is referred to (Baroni and Giacomin 2009; Baroni, Caminada, and Giacomin 2011).

Definition 4. *Let $F = (A, R)$ be an AF, then $S \in cf(F)$ is*

- *a* stable *extension (of F), i.e. $S \in stable(F)$, if $S_R^+ = A$;*
- *an* admissible *extension, i.e. $S \in adm(F)$, if each $a \in S$ is defended by S;*
- *a* preferred *extension, i.e. $S \in prf(F)$, if $S \in adm(F)$ and for each $T \in adm(F)$, $S \not\subset T$;*
- *the* grounded *extension (of F), i.e. $S = grd(F)$, if it is the least fixed-point of the characteristic function $\mathcal{F}_F$;*
- *a* stage *extension (of F), i.e. $S \in stg(F)$, if for each $T \in cf(F)$, $S_R^+ \not\subset T_R^+$.*

Next we consider *cf2* semantics, which is based on a decomposition along the strongly connected components (SCCs) of an AF depending on a given set S of arguments. For a detailed discussion on the *cf2* semantics we refer to (Baroni, Giacomin, and Guida 2005; Baroni and Giacomin 2009; Gaggl and Woltran 2012). We require some further formal machinery and concepts from graph theory. By $SCCs(F)$, we denote the set of *strongly connected components* of an AF $F = (A, R)$, i.e. sets of vertices of the maximal strongly connected[1] sub-graphs of F; $SCCs(F)$ is thus a partition of A. Moreover, for an argument $a \in A$, we denote by $C_F(a)$ the component of F where a occurs in, i.e. the (unique) set $C \in SCCs(F)$, such that $a \in C$. AFs $F_1 = (A_1, R_1)$ and $F_2 = (A_2, R_2)$ are called *disjoint* if $A_1 \cap A_2 = \emptyset$. Moreover, the union between (not necessarily disjoint) AFs is defined as $F_1 \cup F_2 = (A_1 \cup A_2, R_1 \cup R_2)$.

It turns out to be convenient to use two different concepts to obtain sub-frameworks of AFs. Let $F = (A, R)$ be an AF and S a set of arguments. Then, $F|_S = ((A \cap S), R \cap (S \times S))$ is the *sub-framework* of F w.r.t. S, and we also use $F - S = F|_{A \setminus S}$. We note the following relation (which we use implicitly later on), for an AF F and sets S, S': $F|_{S \setminus S'} = F|_S - S' = (F - S')|_S$.

We now give the definition of the *cf2* semantics which slightly differs in notation from (but is equivalent to) the original definition in (Baroni, Giacomin, and Guida 2005).

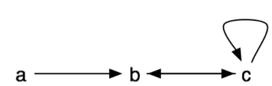
Definition 5. *Let $F = (A, R)$ be an AF and $S \subseteq A$. A $b \in A$ is* component-defeated *by S (in F), if there exists an $a \in S$, s.t. $(a, b) \in R$ and $a \notin C_F(b)$. The set of arguments component-defeated by S in F is denoted by $D_F(S)$.*

Definition 6. *Let $F = (A, R)$ be an argumentation framework and S a set of arguments. Then, S is a cf2 extension of F, i.e. $S \in cf2(F)$, iff*

- *in case $|SCCs(F)| = 1$, then $S \in naive(F)$,*
- *else, $\forall C \in SCCs(F)$, $(S \cap C) \in cf2(F|_C - D_F(S))$.*

In words, an AF is recursively decomposed along its SCCs depending on a set S, where in the base case S needs to be a naive extensions. We illustrate the behavior of the introduced semantics in the following example.

Example 1. *Consider the following AF $F = (A, R)$ with $A = \{a, b, c\}$ and $R = \{(a, b), (b, c), (c, b), (c, c)\}$.*



Then, the above defined semantics yield the following extensions. $stable(F) = \emptyset$; $adm(F) = \{\{\}, \{a\}\}$; $prf(F) =$

[1]A directed graph (an AF) is called *strongly connected* if there is a path from each vertex to every other vertex of the graph.

$grd(F) = \{\{a\}\}$*; and* $naive(F) = stg(F) = \{\{a\}, \{b\}\}$. $S = \{a\}$ *is the only cf2 extension of* F*, as* F *has two SCCs* $C_1 = \{a\}$ *and* $C_2 = \{b, c\}$ *and* $D_F(S) = \{b\}$*. Then,* $(S \cap C_1) \in cf2(F|_{C_1})$ *holds as* $\{a\} \in naive(F|_{C_1})$*, and* $(S \cap C_2) \in cf2(F|_{C_2} - \{b\})$ *holds as* $\emptyset \in naive(F|_{\{c\}})$*. Regarding stage semantics,* $S = \{b\}$ *is a stage extension as* $S^+_R = \{b, c\}$ *and there is not* $T \in cf(F)$ *s.t.* $T^+_R \supset S^+_R$*.* ♢

Properties of *cf2* and Stage Semantics

To avoid the recursive computation of sub-frameworks, Gaggl and Woltran (2010) introduced an alternative characterization of *cf2* semantics which requires the following concepts. The motivation for this was to design a compact Answer-set Programming (ASP) encoding which has also been incorporated in the system ASPARTIX[2] (Egly, Gaggl, and Woltran 2010). Furthermore, it facilitated the analysis of redundant patterns w.r.t. *cf2* semantics (Gaggl and Woltran 2011; 2012) and the proof of general complexity results for reasoning problems regarding the *cf2* semantics (Gaggl and Woltran 2012).

The first concept describes that an AF is separated if there are no attacks between different SCCs and the separation of an AF deletes all attacks between different SCCs.

Definition 7. *An AF* $F = (A, R)$ *is called* separated *if for each* $(a, b) \in R$*,* $C_F(a) = C_F(b)$*. We define* $[[F]] = \bigcup_{C \in SCCs(F)} F|_C$ *and call* $[[F]]$ *the* separation *of* F*.*

Next we consider a restricted reachability relation identifying whether there is a path from an argument to another only using arguments in a specific set B.

Definition 8. *Let* $F = (A, R)$ *be an AF, arguments* $a, b \in A$ *and* $B \subseteq A$*. We say that* b *is* reachable *in* F *from* a modulo B*, in symbols* $a \Rightarrow^B_F b$*, if there exists a path from* a *to* b *in* $F|_B$*, i.e. there exists a sequence* $c_1, \dots, c_n$ $(n > 1)$ *of arguments such that* $c_1 = a$*,* $c_n = b$*, and* $(c_i, c_{i+1}) \in R \cap (B \times B)$*, for all* i *with* $1 \leq i < n$*.*

The operator $\Delta_{F,S}(.)$ (applied to $D = \emptyset$) computes recursively all arguments which are attacked by the set S and can not reach their attacker without going over arguments already in $\Delta_{F,S}(.)$.

Definition 9. *For an AF* $F = (A, R)$*,* $D \subseteq A$ *and* $S \subseteq A$*,*

$$\Delta_{F,S}(D) = \{a \in A \mid \exists b \in S : b \neq a, (b, a) \in R, a \not\Rightarrow^{A \setminus D}_F b\}.$$

$\Delta_{F,S}(.)$ is monotonic and thus it has a least fixed-point (lfp). With slightly abuse of notation we will denote the least fixed-point as $\Delta_{F,S}$.

Now the *cf2* extensions can be characterized as follows.

Proposition 1. *For any AF* F*,*

$$cf2(F) = \{S \mid S \in naive(F) \cap naive([[F - \Delta_{F,S}]])\}.$$

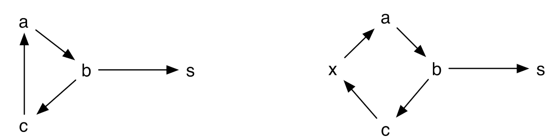
In the following we illustrate how the characterization of Proposition 1 can be used for identifying *cf2* extensions, for a more detailed explanation we refer to (Gaggl and Woltran 2010; 2012).

[2]See *http://rull.dbai.tuwien.ac.at:8080/ASPARTIX/* for a web front-end.

Example 2. *To exemplify the behavior of* $\Delta_{F,S}$ *and* $[[F - \Delta_{F,S}]]$ *let us consider the AF* F *of Example 1.* F *has two naive sets, namely* $S = \{a\}$ *and* $T = \{b\}$*. First, we concentrate on the set* S *and compute* $\Delta_{F,S} = \{b\}$ *and* $[[F - \Delta_{F,S}]] = (\{a, c\}, \{(c, c)\})$*. Thus,* $S \in naive([[F - \Delta_{F,S}]])$ *and clearly* $S \in cf2(F)$*.*
For T *we obtain* $\Delta_{F,T} = \emptyset$ *and* $[[F - \Delta_{F,T}]] = (A, \{(b, c), (c, b), (c, c)\})$*. Now,* $T \notin naive([[F - \Delta_{F,T}]])$*, as there is the set* $T' = \{a, b\} \supset T$ *and* $T' \in cf([[F - \Delta_{F,T}]])$*.*

Now, we focus on the special behavior of *cf2* and stage semantics. They are both based on naive sets, thus they are, in contrast to admissible-based semantics, capable to select arguments out of odd-length cycles as accepted. Consider the following example.

Example 3. *Suppose there are three witnesses* A*,* B *and* C*, where* A *states that* B *is unreliable,* B *states that* C *is unreliable and* C *states that* A *is unreliable. Moreover,* C *has a statement* S*. The graph of the framework* F *is illustrated on the left side. Any admissible-based semantics returns the empty set as its only extension. But if we have four rather than three witnesses, let's call the fourth one* X*, as in the AF* G *on the right side, the situation changes, and the preferred extensions of* G *are* $\{a, c, s\}$ *and* $\{b, x\}$*.*

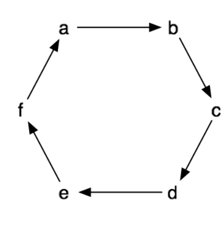


On the other hand, the naive-based semantics return $stg(F) = cf2(F) = \{\{b\}, \{a, s\}, \{c, s\}\}$ *and* $stg(G) = cf2(G) = \{\{a, c, s\}, \{b, x\}\}$*.* ♢

The motivation behind selecting arguments out of an odd-length cycle is to see the arguments as different choices and to be able to choose between them. There is no need for defense, and the naive sets ensure I-maximality (Baroni and Giacomin 2007). A special case of odd-length cycles are self-attacking arguments. One might think that it is not necessary to defend against those "broken" arguments. But, admissible-based semantics are not able to distinguish if it is necessary to defend against an attack or not. In this case it might also be desired to abandon defense and take the naive sets as the basic requirement.

So far, we only discussed the positive behavior of the *cf2* semantics, but unfortunately there are also some disadvantages.

Example 4. *Consider the AF* F*:*



The framework consists of a single SCC, so we obtain $cf2(F) = naive(F) = \{\{a, d\}, \{b, e\}, \{c, f\}, \{a, c, e\},$

$\{b, d, f\}\}$. *In this example we have an even-length cycle and the cf2 semantics produce some strange extensions like* $\{a, d\}, \{b, e\}$ *and* $\{c, f\}$. *Every argument of those extensions is contained in one of the other two cf2 extensions* $\{a, c, e\}$ *and* $\{b, d, f\}$, *hence it does not make much sense to also accept the extensions which defend their attacker. In contrast, the stage extensions are* $stg(F) = \{\{a, c, e\}, \{b, d, f\}\}$, *as expected.* ♢

Example 4 shows, that also the *cf2* semantics has some drawbacks. Furthermore, for AFs F with odd-length cycles ≥ 9, we can also obtain $cf2(F) \neq stg(F)$. Whereas, stage semantics gives more reasonable results especially on single SCCs and still guarantees a uniform treatment of odd-, and even-length cycles. As stage semantics extends stable semantics in the sense that both semantics coincide if at least one stable extension exists, it holds that for SCCs without odd-length cycles stage semantics proposes stable extensions. Similar observations have also been made in (Gabbay 2012). However, for a stage extension it might be the case that even unattacked arguments are not accepted and more general, the grounded extension is not contained in every stage extension. For instance consider the AF F in Example 1 where $grd F = \{a\}$ but $\{b\}$ being a stage extension not containing the unattacked argument a.

Combining Stage and *cf2* Semantics

In the previous section, we observed that the stage semantics has a more intuitive behavior on single SCCs, because there *cf2* semantics only selects the naive extensions.

Our suggestion is to combine the two semantics, where we use the SCC-recursive schema of the *cf2* semantics and instantiate the base case with stage semantics. To retain the naming introduced by Baroni, Giacomin, and Guida (2005), we denote the obtained semantics as *stage2*.

Definition 10. *Let* $F = (A, R)$ *be an AF and* $S \subseteq A$. *Then,* S *is a stage2 extension of* F, *i.e.* $S \in stage2(F)$, *iff*

- *in case* $|SCCs(F)| = 1$, *then* $S \in stg(F)$,
- *else,* $\forall C \in SCCs(F)$, $(S \cap C) \in stage2(F|_C - D_F(S))$.

According to the alternative characterization of *cf2* semantics, one can also formulate *stage2* semantics in the same way.

Proposition 2. *For any AF* F,

$$stage2(F) = \{S \mid S \in naive(F) \cap stg([[F - \Delta_{F,S}]])\}.$$

The remainder of this section is devoted to the proof of Proposition 2. To show that the alternative characterization is equivalent to Definition 10, we need to define two more formal concepts. First, we define the set of recursively component defeated arguments $\mathcal{RD}_F(S)$ as in (Gaggl and Woltran 2010).

Definition 11. *Let* $F = (A, R)$ *be an AF and* $S \subseteq A$. *We define the set of arguments* recursively component defeated *by* S *(in* F*) as follows:*

- *if* $|SCCs(F)| = 1$ *then* $\mathcal{RD}_F(S) = \emptyset$; *else,*
- $\mathcal{RD}_F(S) = D_F(S) \cup \bigcup_{C \in SCCs(F)} \mathcal{RD}_{F|_C - D_F(S)}(S \cap C)$.

Next, we define the level of recursiveness a framework shows with respect to a set S of arguments.

Definition 12. *For an AF* $F = (A, R)$ *and* $S \subseteq A$, *we recursively define the* level $\ell_F(S)$ *of* F *w.r.t.* S *as follows:*

- *if* $|SCCs(F)| = 1$ *then* $\ell_F(S) = 1$;
- *otherwise,* $\ell_F(S) = 1 + max(\{\ell_{F|_C - D_F(S)}(S \cap C) \mid C \in SCCs(F)\})$.

Lemma 1. *For any AF* $F = (A, R)$, $S \subseteq A$. *Let* $\mathcal{R}'_{F,C,S} = \mathcal{RD}_{F|_C - D_F(S)}(S \cap C)$, *then*

$$(F|_C - D_F(S)) - \mathcal{R}'_{F,C,S} = F|_C - \mathcal{RD}_F(S).$$

Proof. The observation has been proven in more detail in (Gaggl and Woltran 2010). Here we just sketch the idea. We fix a $C \in SCCs(F)$. Since for each further $C' \in SCCs(F)$ (i.e. $C \neq C'$), no argument from $\mathcal{RD}_{F|_{C'} - D_F(S)}(S \cap C')$ occurs in $F|_C$, the assertion follows. □

Lemma 2 gives the first alternative characterization of *stage2*.

Lemma 2. *Let* $F = (A, R)$ *be an AF and* $S \subseteq A$. *Then,*

$$S \in stage2(F) \text{ iff } S \in stg([[F - \mathcal{RD}_F(S)]]).$$

Proof. We show the claim by induction over $\ell_F(S)$.

Induction base. For $\ell_F(S) = 1$, we have $|SCCs(F)| = 1$. By definition $\mathcal{RD}_F(S) = \emptyset$ and we have $[[F - \mathcal{RD}_F(S)]] = [[F]] = F$. Thus, the assertion states that $S \in stage2(F)$ iff $S \in stg(F)$ which matches the original definition for the *stage2* semantics in case the AF has a single strongly connected component.

Induction step. Let $\ell_F(S) = n$ and assume the assertion holds for all AFs F' and sets S' with $\ell_{F'}(S') < n$. In particular, we have by definition that, for each $C \in SCCs(F)$, $\ell_{F|_C - D_F(S)}(S \cap C) < n$. By the induction hypothesis and Lemma 1, we thus obtain that, for each $C \in SCCs(F)$ the following holds:

$$\begin{aligned} &(S \cap C) \in stage2(F|_C - D_F(S)) \text{ iff} \\ &(S \cap C) \in stg\big([[F|_C - \mathcal{RD}_F(S)]]\big). \end{aligned} \tag{1}$$

We now prove the assertion. Let $S \in stage2(F)$. By definition, for each $C \in SCCs(F)$, $(S \cap C) \in stage2(F|_C - D_F(S))$. Using (1), we get that for each $C \in SCCs(F)$, $(S \cap C) \in stg([[F|_C - \mathcal{RD}_F(S)]])$. By the definition of components and the semantics of stage, the following relation thus follows:

$$\bigcup_{C \in SCCs(F)} (S \cap C) \in stg\Big(\bigcup_{C \in SCCs(F)} [[F|_C - \mathcal{RD}_F(S)]]\Big).$$

Since $S = \bigcup_{C \in SCCs(F)} (S \cap C)$ and due to (Gaggl and Woltran 2010), $\bigcup_{C \in SCCs(F)} [[F|_C - \mathcal{RD}_F(S)]] = [[F - \mathcal{RD}_F(S)]]$, we arrive at $S \in stg([[F - \mathcal{RD}_F(S)]])$ as desired. The other direction is by essentially the same arguments. □

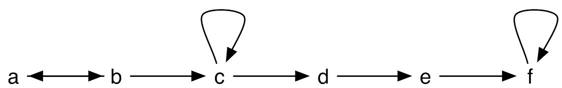
Proof of Proposition 2. The result holds by the following observations. By Lemma 2, $S \in stage2(F)$ iff $S \in stg([[F - \mathcal{RD}_F(S)]])$. Moreover, due to (Gaggl and Woltran 2010), for any $S \in cf(F)$, $\Delta_{F,S} = \mathcal{RD}_F(S)$. Finally, $S \in stage2(F)$ implies $S \in naive(F)$. □

We obtain for the framework F of Example 1, $stage2(F) = cf2(F) = \{\{a\}\}$, and for the AF of Example 4, $stage2(F) = stg(F) = \{\{a, c, e\}, \{b, d, f\}\}$.

Comparison of *stage2* with other Semantics

Here we compare semantics w.r.t. the ⊆-relations between the sets of extensions.

First, in general stage and *stage2* semantics are incomparable w.r.t. set inclusion. For instance, consider the following AF F.



Then, $stage2(F) = \{\{a, d\}, \{b, d\}\}$, but $stg(F) = \{\{b, d\}, \{b, e\}\}$.

Now, we consider the relation between *cf2* and *stage2* semantics. By Example 4 we know that there are AFs with $cf2(F) \not\subseteq stage2(F)$.

Proposition 3. *For any AF* $F = (A, R)$*,* $stage2(F) \subseteq cf2(F)$*.*

Proof. Consider a set $S \in stage2(F)$. By Proposition 2, $S \in naive(F) \cap stg([[F - \Delta_{F,S}]])$. Now using that for every AF G, $stg(G) \subseteq naive(G)$ we obtain $S \in naive(F) \cap naive([[F - \Delta_{F,S}]])$. By Proposition 1, $S \in cf2(F)$. □

Next, we study the relations between stable and *stage2* semantics.

Proposition 4. *For any AF* $F = (A, R)$*,* $stable(F) \subseteq stage2(F)$*.*

Proof. Consider $E \in stable(F)$, then we know that $E \in naive(F)$ and for each $a \in A \setminus E$ there exists $b \in E$ such that $(b, a) \in R$. Hence, $a \in E^+_{R_F}$. It remains to show that $E \in stg([[F - \Delta_{F,E}]])$. We show the stronger statement $E \in stable([[F - \Delta_{F,E}]])$.

To this end, let $F' = F - \Delta_{F,E}$ and $F'' = [[F - \Delta_{F,E}]]$, we have either $a \in \Delta_{F,E}$ or $a \in A_{F'}$. For $a \in A_{F'} = A_{F''}$, we need to show that $a \in E^+_{R_{F''}}$. If $a \in E$ clearly $a \in E^+_{R_{F''}}$, hence we consider $a \in A_{F'} \setminus E$. As E is stable there exists $b \in E$ such that $(b, a) \in R_{F'}$. Now as $a \notin \Delta_{F,E}$, by Definition 9 we know that $a \Rightarrow_F^{A \setminus \Delta_{F,E}} b$. In other words a, b are in the same SCC of F' and thus $(b, a) \in R_{F''}$. Hence, for every $a \in A_{F''} \setminus E$ there is an argument $b \in E$ such that $(b, a) \in R_{F''}$, hence $E \in stable(F'')$. As for any AF G $stable(G) \subseteq stg(G)$, it follows that $E \in stg(F'')$. Thus, by Proposition 2, $E \in stage2(F)$. □

Figure 1 gives an overview of the relations between naive-based semantics. An arrow from semantics σ to semantics τ

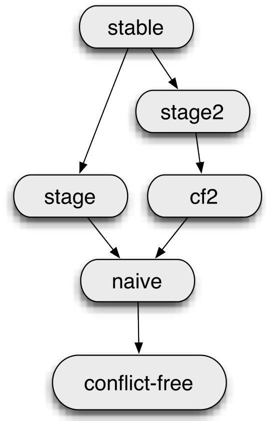
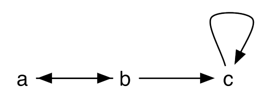


Figure 1: Relations between naive-based semantics

encodes that each σ-extension is also a τ-extension. Furthermore, if there is no directed path from σ to τ, then one can construct AFs with a σ-extension that is not a τ-extension.

If an AF possesses at least one stable extension, stage coincides with stable semantics. Obviously, this does not hold for *stage2* semantics, for instance consider the AF $F = (\{a, b, c\}, \{(a, b), (b, a), (b, c), (c, c)\})$.



We obtain $stage2(F) = \{\{a\}, \{b\}\}$ and $stable(F) = \{\{b\}\}$. However, these semantics comply with each other in *coherent* AFs, i.e. AFs where stable and preferred semantics coincide.

Proposition 5. *For any coherent AF* F*,* $stable(F) = stg(F) = stage2(F)$*.*

Proof. By Proposition 4, $stable(F) \subseteq stage2(F)$ and thus it only remains to show that also $stable(F) \supseteq stage2(F)$ holds.

Let us first consider the case where F consists of a single SCC. Then, *stage2* semantics coincides with stage semantics and as F is coherent also with stable semantics.

Now, let this be our induction base, and let us assume the claim holds for AFs of size $< n$. Let us consider an AF F of size n with $(C_i)_{1 \leq i \leq m}$ being the SCCs of F, such that there is no attack from C_i to C_j for $j < i$. If $m = 1$ we are in the base-case, hence let us assume that $m \geq 2$. Consider $S \in stage2(F)$ and $S_1 = S \cap \bigcup_{1 \leq i < m} C_i$, $S_2 = S \cap C_m$. By definition of *stage2* we know that $S_1 \in stage2(F - C_m)$ and $S_2 \in stage2(F|_{C_m} - S_1^+)$. Note, $S_1 \cap S_2 = \emptyset$.

By assumption, F is coherent and it is easy to see that also $F - C_m$ is coherent. Hence, by the induction hypothesis, $stable(F - C_m) = prf(F - C_m) = stage2(F - C_m)$.

Next, we show that also $F|_{C_m} - S_1^+$ is coherent. By definition, $stable(F) \subseteq prf(F)$. Now, consider an extension $E_2 \in prf(F|_{C_m} - S_1^+)$. By the directionality of prf and the fact that $S_1 \in stable(F - C_m)$, we obtain $(S_1 \cup E_2) \in prf(F)$. Now, as F is coherent also $(S_1 \cup E_2) \in stable(F)$ and thus, $E_2 \in stable(F|_{C_m} - S_1^+)$. Hence, $F|_{C_m} - S_1^+$ is coherent and again we can use the induction hypothesis.

Finally, we obtain $S_1 \in stable(F - C_m)$ and $S_2 \in stable(F|_{C_m} - S_1^+)$, combining these results we get $S \in stable(F)$. □

Notice, the last theorem implies that on coherent AFs $stage2$ semantics coincides with preferred, stage and semi-stable (Caminada, Carnielli, and Dunne 2011) semantics, because on coherent AFs all these semantics coincide with stable semantics.

Extension Evaluation Criteria

Several general criteria for the evaluation of argumentation semantics have been proposed in (Baroni and Giacomin 2007). In this subsection we analyze the criteria relevant for naive-based semantics.

Definition 13. *A semantics σ satisfies*

- *the* I-maximality criterion *if for each AF $F = (A, R)$, and for each $S_1, S_2 \in \sigma(F)$, if $S_1 \subseteq S_2$ then $S_1 = S_2$;*
- *the* reinstatement criterion *if for each AF $F = (A, R)$, and for each $S \in \sigma(F)$, a defended by S implies $a \in S$.*
- *the* weak reinstatement criterion*, if for each $F = (A, R)$, and for each $S \in \sigma(F), E \in grd(F) : E \subseteq S$;*
- *the* $\mathcal{CF}$-reinstatement criterion*, if for each $F = (A, R)$, for each $S \in \sigma(F), \forall b : (b, a) \in R, \exists c \in S : (c, b) \in R$ and $S \cup \{a\} \in cf(F) \Rightarrow a \in S$.*
- *the* directionality criterion *if for each $F = (A, R)$, and for each set of unattacked arguments $U \subseteq A$ (s. t. $\forall a \in A \setminus U$ there is no $b \in U$ with $(a, b) \in R$), it holds that $\sigma(F|_U) = \{(S \cap U) \mid S \in \sigma(F)\}$.*

We start with some general properties of naive-based semantics.

Proposition 6. *I-maximality and $\mathcal{CF}$-reinstatement are satisfied by each semantics σ with $\sigma(F) \subseteq naive(F)$.*

Proof. Clearly $naive$ semantics satisfies both I-maximality and $\mathcal{CF}$-reinstatement. A set E which is $\subseteq$-maximal in $naive(F)$ is also maximal in each subset of $naive(F)$ and thus, σ satisfies I-maximality. $\mathcal{CF}$-reinstatement is a property defined on single extensions, and as each σ-extension is also a $naive$ extension, $\mathcal{CF}$-reinstatement is satisfied . □

Among the naive-based semantics, only stable semantics satisfies the reinstatement property, which is due to the fact that it is also an admissible-based semantics.

Proposition 7. *The reinstatement property is not satisfied by semantics which can select non-empty conflict-free subsets out of odd-length cycles.*

Proof. Consider an odd length cycle $F = (\{a_1, \dots, a_n\}, \{(a_i, a_{i+1 \bmod n}) \mid 1 \leq i \leq n\})$ with n being an odd integer. We claim that no $E \in cf(F)$ and $E \neq \emptyset$ satisfies the reinstatement property. Now, towards a contradiction let us assume there exists a nonempty $E \in cf(F)$ satisfying the reinstatement property. W.l.o.g. assume that $a_1 \in E$. Then a_3 is defended and by assumption $a_3 \in E$. But then also a_5 is defended, and by induction it follows that $a_i \in E$ if i is odd. Hence also $a_n \in E$, but $\{a_1, a_n\} \subseteq E$ contradicts that E is conflict-free in F. □

Hence, when considering naive-based semantics we are usually interested in weaker forms of reinstatement, namely the weak- or $\mathcal{CF}$-reinstatement.

Proposition 8. *The weak reinstatement and directionality criterion are not satisfied by naive and stage semantics.*

Proof. Consider the AF F from Example 1. We obtain $naive(F) = stg(F) = \{\{a\}, \{b\}\}$ and the grounded extension $G = \{a\}$. Then, the weak reinstatement criterion is not satisfied because $G \not\subseteq \{b\}$. Now let us consider directionality and the sub-framework $F|_{\{a\}}$. Then $stg(F|_{\{a\}}) = \{\{a\}\}$ but $\{(\{a\} \cap S) \mid S \in stg(F)\} = \{\emptyset, \{a\}\}$, contradicting the directionality criterion. □

Proposition 9. *The weak reinstatement criterion is satisfied by $stage2$ semantics.*

Proof. Let $F = (A, R)$ and $E \in grd(F)$. Due to (Baroni, Giacomin, and Guida 2005), for any AF F and any $S \in cf2(F)$, $E \subseteq S$. From Proposition 3 we know that for any AF G, $stage2(G) \subseteq cf2(G)$. It follows that for any extension $S \in stage2(F)$, $S \in cf2(F)$ and $E \subseteq S$. □

We sum up the results for the novel $stage2$ semantics.

- Directionality is satisfied. Due to (Baroni and Giacomin 2007), any SCC-recursive semantics σ satisfies the directionality criterion. As the $stage2$ semantics has been directly defined in terms of the SCC-recursive schema, the directionality criterion is indeed satisfied.
- I-maximality and $\mathcal{CF}$-reinstatement are satisfied, see Proposition 6.
- Reinstatement is not satisfied, see Proposition 7.
- Weak reinstatement is satisfied, see Proposition 9.

We summarize the evaluation criteria w.r.t. naive-based semantics in Table 1.

Finally, we mention that directionality implies the properties crash-resistance and non-interference (cf. (Baroni, Caminada, and Giacomin 2011)) which both are violated by stable semantics, but satisfied by $stage2$.

	$naive$	$stable$	stg	$cf2$	$stage2$
I-max.	Yes	Yes	Yes	Yes	Yes
Reinst.	No	Yes	No	No	No
Weak reinst.	No	Yes	No	Yes	Yes
$\mathcal{CF}$-reinst.	Yes	Yes	Yes	Yes	Yes
Direct.	No	No	No	Yes	Yes

Table 1: Evaluation Criteria w.r.t. Naive-based Semantics.

Redundant Patterns w.r.t. *stage2* Semantics

Recently, redundant patterns for AFs w.r.t. specific semantics have been studied. In (Amgoud and Vesic 2011) the notion of equivalence w.r.t. stable semantics has been studied for logic-based argumentation systems. Whereas, Oikarinen and Woltran (Oikarinen and Woltran 2011) identified kernels which eliminate redundant attacks of AFs and introduced the concept of strong equivalence as follows.

Definition 14. *Two AFs F and G are* strongly equivalent *to each other w.r.t. a semantics σ, in symbols $F \equiv_s^\sigma G$, iff for each AF H, $\sigma(F \cup H) = \sigma(G \cup H)$.*

By definition, $F \equiv_s^\sigma G$ implies $\sigma(F) = \sigma(G)$, but the other direction is not true in general. In (Gaggl and Woltran 2011; 2012), it has been shown that for *cf2* semantics, strong equivalence coincides with syntactic equivalence. In other words, there are no redundant patterns at all. In the following, we show that the same holds for *stage2* semantics as well.

Theorem 1. *For any AFs F and G, $F \equiv_s^{stage2} G$ iff $F = G$.*

Proof. Since for any AFs $F = G$ obviously implies for all AFs H, $stage2(F \cup H) = stage2(G \cup H)$, we only have to show that if $F \neq G$ there exists an AF H such that $stage2(F \cup H) \neq stage2(G \cup H)$.
For any two AFs F and G, strong equivalence w.r.t. naive-based semantics requires that the AFs coincide with the arguments and the self-attacks (Gaggl and Woltran 2011). We thus assume that $A = A_F = A_G$ and $(a, a) \in R_F$ iff $(a, a) \in R_G$, for each $a \in A$. Let us thus suppose w.l.o.g. an attack $(a, b) \in R_F \setminus R_G$ and consider the AF

$$\begin{aligned} H \;=\; & (A \cup \{d, x, y, z, z1\}, \{(a, a), (b, b), (b, x), (x, a), \\ & (a, y), (y, z), (z, a), (z, z1), (z1, z), (z1, z1), \\ & (d, c) \mid c \in A \setminus \{a, b\}\}), \end{aligned}$$

see also Figures 2 and 3 for illustration.

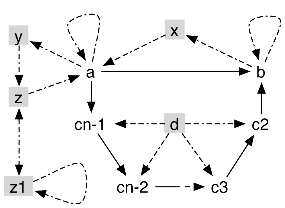


Figure 2: $F \cup H$

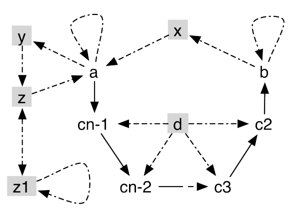


Figure 3: $G \cup H$

Then, there exists a set $E = \{d, x, z\}$, such that $E \in stage2(F \cup H)$ but $E \notin stage2(G \cup H)$. To show that $E \in stage2(F \cup H)$, we first compute $\Delta_{F \cup H, E} = \{c \mid c \in A \setminus \{a, b\}\}$. Thus, in the instance $F' = [[(F \cup H) - \Delta_{F \cup H, E}]]$ we have two SCCs left, namely $C_1 = \{d\}$ and $C_2 = \{a, b, x, y, z, z1\}$ as illustrated in Figure 4. Furthermore, all attacks between the arguments of C_2 are preserved, and we obtain that $E \in stg(F')$, and as E is also a naive set of $(F \cup H)$, $E \in stage2(F \cup H)$ follows. On the other hand, we obtain $\Delta_{G \cup H, E} = \{a\} \cup \{c \mid c \in A \setminus \{a, b\}\}$, and the instance $G' = [[(G \cup H) - \Delta_{G \cup H, E}]]$ consists of five SCCs, namely $C_1 = \{d\}$, $C_2 = \{b\}$, $C_3 = \{x\}$, $C_4 = \{y\}$ and $C_5 = \{z, z1\}$, with b and $z1$ being self-attacking as illustrated in Figure 5.

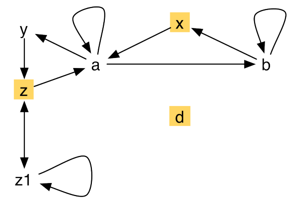


Figure 4: F'

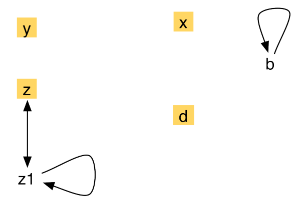


Figure 5: G'

Thus, the set $T = \{d, x, y, z\} \supset E$ is conflict-free in G' and $T^+_{R_{G'}} \supset E^+_{R_{G'}}$. Therefore, we obtain $E \notin stg(G')$, and hence, $E \notin stage2(G \cup H)$. $F \not\equiv_s^{stage2} G$ follows. □

No matter which AFs $F \neq G$ are given, we can always construct a framework H such that $stage2(F \cup H) \neq stage2(G \cup H)$. In particular, we can always add new arguments and attacks such that the missing attack in one of the original frameworks leads to different SCCs in the modified ones and therefore to different *stage2* extensions, when suitably augmenting the two AFs under comparison. Till now, *stage2* is the second semantics beside *cf2*, where strong equivalence coincides with syntactic equivalence.

To identify to which extent attacks contribute in terms of a given semantics, the *succinctness property* has been introduced in (Gaggl and Woltran 2012). In contrast to strong equivalence which considers particular AFs, the succinctness property denotes a general property for argumentation semantics. Hence, it is independent of the specific instantiation method.

Before we give the definition of the succinctness property, we define what we mean with redundant attacks; for AFs $F = (A, R)$ and $F' = (A', R')$ we write $F \subseteq F'$ to denote that $A \subseteq A'$ and $R \subseteq R'$ jointly hold. Moreover, we use $F \setminus (a, b)$ as a shorthand for the framework $(A, R \setminus \{(a, b)\})$.

Definition 15. *For an AF $F = (A, R)$ and semantics σ we call an attack $(a, b) \in R$* redundant *in F* w.r.t. *σ if for all F' with $F \subseteq F'$, $\sigma(F') = \sigma(F' \setminus (a, b))$.*

Definition 16. *An argumentation semantics σ satisfies the* succinctness property *or is* maximal succinct *iff no AF contains a redundant attack w.r.t. σ.*

The following proposition gives the link between the succinctness property and strong equivalence.

Proposition 10. *(Gaggl and Woltran 2012) An argumentation semantics σ satisfies the succinctness property iff for any AFs F, G with $A_F = A_G$: ($F \equiv_s^\sigma G \Leftrightarrow F = G$).*

We point out that for all semantics considered so far, strong equivalence for AFs implies that the AFs have the same arguments. Thus, for the semantics under our consideration, one can drop the condition $A_F = A_G$ in the above proposition.

From Theorem 1 and Proposition 10 we conclude that the succinctness property is satisfied by *stage2* semantics.

Computational Complexity

In this section, we turn to computational issues. We assume the reader has knowledge about standard complexity classes, i.e. P, NP and coNP. Nevertheless, we briefly recapitulate the concept of oracle machines and some related complexity classes. Let $\mathcal{C}$ notate some complexity class. By a $\mathcal{C}$-oracle machine we mean a (polynomial time) Turing machine which can access an oracle that decides a given (sub)-problem in $\mathcal{C}$ within one step. We denote the class of decision problems, that can be solved by such machines, as $\mathrm{P}^{\mathcal{C}}$ if the underlying Turing machine is deterministic and $\mathrm{NP}^{\mathcal{C}}$ if the underlying Turing machine is non-deterministic. The class $\Sigma_2^{\mathrm{P}} = \mathrm{NP}^{\mathrm{NP}}$ denotes problems which can be decided by a non-deterministic polynomial time algorithm that has access to an NP-oracle. The class $\Pi_2^{\mathrm{P}} = \mathrm{coNP}^{\mathrm{NP}}$ is defined as the complementary class of Σ_2^{P}, i.e. $\Pi_2^{\mathrm{P}} = \mathrm{co}\Sigma_2^{\mathrm{P}}$. Finally, we give an overview of the relations between the introduced complexity classes.

$$\mathrm{P} \subseteq \begin{array}{c}\mathrm{NP}\\ \mathrm{coNP}\end{array} \subseteq \begin{array}{c}\Sigma_2^{\mathrm{P}}\\ \Pi_2^{\mathrm{P}}\end{array}$$

The typical reasoning problems in abstract argumentation are the following (for a semantics σ):

- Cred_σ: Given AF $F = (A, R)$ and $a \in A$. Is a contained in *some* $S \in \sigma(F)$?
- Skept_σ: Given AF $F = (A, R)$ and $a \in A$. Is a contained in *each* $S \in \sigma(F)$?
- Ver_σ: Given AF $F = (A, R)$ and $S \subseteq A$. Is $S \in \sigma(F)$?

The complexity of these problems for different semantics is well studied in the literature (see e.g. (Dunne and Wooldridge 2009)). Next, we provide a complexity analysis for *stage2* semantics, exploiting the corresponding results for stage semantics (Dvořák and Woltran 2010).

Theorem 2. *For stage2 semantics the following holds*

- $\mathit{Cred}_{\mathit{stage2}}$ *is* Σ_2^{P}*-complete.*
- $\mathit{Skept}_{\mathit{stage2}}$ *is* Π_2^{P}*-complete.*
- $\mathit{Ver}_{\mathit{stage2}}$ *is* coNP*-complete.*

Proof. We first consider the membership part starting with $\mathit{Ver}_{\mathit{stage2}}$. Given an AF $F = (A, R)$ a set E of arguments by Proposition 2 we have to check whether $E \in \mathit{naive}(F)$ (which can be done in P), and whether $E \in \mathit{stg}([[F - \Delta_{F,S}]])\}$. As $[[F - \Delta_{F,S}]]$ can be constructed in polynomial time and $\mathit{Ver}_{\mathit{stg}} \in \mathrm{coNP}$, the latter is in coNP and thus also $\mathit{Ver}_{\mathit{stage2}} \in \mathrm{coNP}$. The problems $\mathit{Cred}_{\mathit{stage2}}$ and $\mathit{Skept}_{\mathit{stage2}}$ can be solved by a standard guess and check algorithm, i.e. guessing an extension containing the argument (resp. not containing the argument) and using an NP-oracle to verify the extension.

For the hardness part we give a reduction $\mathcal{R}$ mapping argumentation frameworks to argumentation frameworks, such that for each AF F it holds that $\mathit{stg}(F) = \mathit{stage2}(\mathcal{R}(F))$[3]. The hardness results then follow from the corresponding hardness results for stage semantics (Dvořák and Woltran 2010).

Given an AF $F = (A, R)$ we define $\mathcal{R}(F) = (A^*, R^*)$ with $A^* = A \cup \{t\}$ and $R^* = R \cup \{(t, t)\} \cup \{(t, a), (a, t) \mid a \in A\})$, where t is a fresh argument. Then, $\mathcal{R}(F)$ has just a single SCC and hence $\mathit{stg}(\mathcal{R}(F)) = \mathit{stage2}(\mathcal{R}(F))$. It remains to show that $\mathit{stg}(F) = \mathit{stg}(\mathcal{R}(F))$. First, as $(t, t) \in R^*$, the argument t can not be contained in a stage extensions. Furthermore, the reduction $\mathcal{R}$ does not modify attacks between argument in A we obtain $\mathit{cf}(F) = \mathit{cf}(\mathcal{R}(F))$. By the construction of $\mathcal{R}(F)$, for each non-empty $E \subseteq A$ $E_R^+ \cup \{t\} = E_{R^*}^+$ thus, $\mathit{stg}(F) = \mathit{stg}(\mathcal{R}(F))$. It is easy to see that $\emptyset \in \mathit{stg}(F)$ iff $\mathit{cf}(F) = \{\emptyset\}$ iff $\emptyset \in \mathit{stg}(\mathcal{R}(F))$. □

[3] Such a $\mathcal{R}$ is called an exact translation for $\mathit{stg} \Rightarrow \mathit{stage2}$ in (Dvořák and Woltran 2011).

	naive	*stable*	*stg*	*cf2*	*stage2*
Cred_σ	in P	NP-c	Σ_2^{P}-c	NP-c	Σ_2^{P}-c
Skept_σ	in P	coNP-c	Π_2^{P}-c	coNP-c	Π_2^{P}-c
Ver_σ	in P	in P	coNP-c	in P	coNP-c

Table 2: Computational Complexity of naive-based semantics ($\mathcal{C}$-c denotes completeness for class $\mathcal{C}$).

We summarize the complexity results for naive-based semantics in Table 2. The results, for naive semantics are due to (Coste-Marquis, Devred, and Marquis 2005), for stable semantics follows from (Dimopoulos and Torres 1996), for stage semantics have been shown in (Dvořák and Woltran 2010), and the results for *cf2* semantics can be found in (Gaggl and Woltran 2012).

Considering the plethora of argumentation semantics, beside *stage2*, only for stage and semi-stable semantics the complexity of both skeptical and credulous reasoning is located on the second level of the polynomial hierarchy[4]. This indicates that *stage2* is among the computationally hardest semantics but in the same breath also among the most expressive ones.

Conclusion

We discussed the drawbacks of the existing naive-based semantics *cf2* and stage and proposed the new semantics *stage2* which combines concepts of *cf2* and stage to overcome their shortcomings.

We provided a broad discussion of *stage2*, its properties and relations to other semantics. First, beside the definition via the SCC-recursive schema we provided an alternative characterization which is similar to that of *cf2* semantics and thus allows to extend several results for *cf2* also to *stage2*. Further, we showed that *stage2* fixes the shortcomings of stage semantics w.r.t. the extension evaluation criteria proposed by (Baroni and Giacomin 2007). We related *stage2* semantics to the existing semantics showing that $\mathit{stable}(F) \subseteq \mathit{stage2}(F) \subseteq \mathit{cf2}(F)$. Moreover, we

[4] For preferred semantics only skeptical acceptance is located on the second level of the polynomial hierarchy while credulous acceptance is NP-complete (Dunne and Bench-Capon 2002).

observed that on coherent AFs $stage2$ semantics coincides with stable and preferred semantics.

Concerning redundant patterns, it turned out that $stage2$ semantics is the second semantics beside $cf2$, where strong equivalence coincides with syntactic equivalence. This means that there no redundant patterns at all, and $stage2$ semantics satisfies the succinctness property proposed in (Gaggl and Woltran 2012).

Finally, we provided a complexity analysis showing $stage2$ semantics is located at the second level of the polynomial hierarchy and thus among the hardest argumentation semantics. These complexity results guide the way to computationally adequate encodings in target formalism like answer-set programming (Egly, Gaggl, and Woltran 2010) or quantified boolean formulas (Egly and Woltran 2006).

Recently, Dov Gabbay dedicated an article to the equational approach of $cf2$ semantics (Gabbay 2012). Therein, he introduced several new semantics to overcome the problems with $cf2$. We leave a detailed comparison of those semantics with $stage2$ for future work. Furthermore, we identify the following two directions for future work. The first one being a more fine grained analysis of computational issues and appropriate implementations of $stage2$ semantics. The latter concerning the role of $stage2$ in the entire argumentation process. For instance, in (Baroni, Caminada, and Giacomin 2011) it has been shown that both, $cf2$ and stage semantics fail the rationality postulates proposed in (Caminada and Amgoud 2007) when one uses the instantiation method proposed therein. Hence, one question for such investigation would be, under which circumstances $stage2$ semantics satisfies the proposed rationality postulates.

References

Amgoud, L., and Vesic, S. 2011. On the equivalence of logic-based argumentation systems. In Benferhat, S., and Grant, J., eds., *Proc. SUM 2011*, volume 6929 of *LNCS*, 123–136. Springer.

Baroni, P., and Giacomin, M. 2007. On principle-based evaluation of extension-based argumentation semantics. *Artif. Intell.* 171(10-15):675–700.

Baroni, P., and Giacomin, M. 2009. Semantics of abstract argument systems. In Rahwan, I., and Simari, G., eds., *Argumentation in Artificial Intelligence*. Springer. 25–44.

Baroni, P.; Caminada, M.; and Giacomin, M. 2011. An introduction to argumentation semantics. *Knowledge Eng. Review* 26(4):365–410.

Baroni, P.; Giacomin, M.; and Guida, G. 2005. Scc-recursiveness: A general schema for argumentation semantics. *Artif. Intell.* 168(1-2):162–210.

Bench-Capon, T. J. M. 2003. Persuasion in practical argument using value-based argumentation frameworks. *Journal of Logic and Computation* 13(3):429–448.

Bodanza, G. A., and Tohm, F. A. 2009. Two approaches to the problems of self-attacking arguments and general odd-length cycles of attack. *Journal of Applied Logic* 7(4):403 – 420. Special Issue: Formal Models of Belief Change in Rational Agents.

Caminada, M., and Amgoud, L. 2007. On the evaluation of argumentation formalisms. *Artif. Intell.* 171(5-6):286–310.

Caminada, M.; Carnielli, W. A.; and Dunne, P. E. 2011. Semi-stable semantics. *Journal of Logic and Computation*.

Coste-Marquis, S.; Devred, C.; and Marquis, P. 2005. Symmetric argumentation frameworks. In Godo, L., ed., *Proc ECSQARU 2005*, volume 3571 of *LNCS*, 317–328. Springer.

Dimopoulos, Y., and Torres, A. 1996. Graph theoretical structures in logic programs and default theories. *Theor. Comput. Sci.* 170(1-2):209–244.

Dung, P. M. 1995. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artif. Intell.* 77(2):321–358.

Dunne, P. E., and Bench-Capon, T. J. M. 2002. Coherence in finite argument systems. *Artif. Intell.* 141(1/2):187–203.

Dunne, P. E., and Wooldridge, M. 2009. Complexity of abstract argumentation. In Simari, G., and Rahwan, I., eds., *Argumentation in Artificial Intelligence*. Springer US. 85–104.

Dvořák, W., and Woltran, S. 2010. Complexity of semi-stable and stage semantics in argumentation frameworks. *Inf. Process. Lett.* 110(11):425–430.

Dvořák, W., and Woltran, S. 2011. On the intertranslatability of argumentation semantics. *J. Artif. Intell. Res. (JAIR)* 41:445–475.

Egly, U., and Woltran, S. 2006. Reasoning in argumentation frameworks using quantified boolean formulas. In Dunne, P. E., and Bench-Capon, T. J. M., eds., *Proc. COMMA 2006*, volume 144 of *FAIA*, 133–144. IOS Press.

Egly, U.; Gaggl, S.; and Woltran, S. 2010. Answer-set programming encodings for argumentation frameworks. *In Argument and Computation* 1(2):147–177.

Gabbay, D. M. 2012. The equational approach to cf2 semantics. *CoRR* abs/1203.0220.

Gaggl, S. A., and Woltran, S. 2010. cf2 semantics revisited. In Baroni, P.; Cerutti, F.; Giacomin, M.; and Simari, G. R., eds., *Proc. COMMA 2010*, volume 216 of *FAIA*, 243–254. IOS Press.

Gaggl, S. A., and Woltran, S. 2011. Strong equivalence for argumentation semantics based on conflict-free sets. In Liu, W., ed., *Proc. ECSQARU 2011*, volume 6717 of *LNCS*, 38–49. Springer.

Gaggl, S. A., and Woltran, S. 2012. The cf2 argumentation semantics revisited. *Journal of Logic and Computation*; doi: 10.1093/logcom/exs011.

Oikarinen, E., and Woltran, S. 2011. Characterizing strong equivalence for argumentation frameworks. *Artif. Intell.* 175(14-15):1985–2009.

Verheij, B. 1996. Two approaches to dialectical argumentation: admissible sets and argumentation stages. In Meyer, J., and van der Gaag, L., eds., *Proc.NAIC 1996*, 357–368.