Exercise Sheet 1: Relational Algebra

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Database Theory, 2020-04-20, Summer Term 2021

Exercise 1.1. Consider a cinema database with tables of the following form (adapted from a similar example in the textbook of Abiteboul, Hull and Vianu):

Films

Title	*Director*	*Actor*
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
…	…	…
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
…	…	…
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

Venues

Cinema	*Address*	*Phone*
UFA	St. Petersburger Str. 24	4825825
Schauburg	Königsbräcker Str. 55	8032185
CinemaxX	Hüblerstr. 8	3158910
…	…	…

Program

Cinema	*Title*	*Time*
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Express the following queries in relational algebra:

1. Who is the director of "The Imitation Game"?
2. Which cinemas feature "The Imitation Game"?
3. What are the address and phone number of "Schauburg"?
4. Is a film directed by "Smith" playing in some cinema?
5. List the pairs of persons such that the first directed the second in a film and vice versa.
6. List the names of directors who have acted in a film they directed.
7. Always return $\{\mathit{Title} \mapsto \text{"Apocalypse Now"}, \mathit{Director} \mapsto \text{"Coppola"}\}$ as the answer.
8. Find the actors cast in at least one film by "Smith."
9. Find the actors cast only in films by "Smith."

Exercise 1.2. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use $\emptyset$ to denote the empty table with no rows and no columns.

Now for a table R, what are the results of the following expressions?

$$R \bowtie R \qquad R \bowtie \emptyset \qquad R \bowtie \{\varepsilon\}$$

Exercise 1.3. Express the following operations using other operations presented in the lecture:

- Intersection $R \cap S$.
- Cross product (Cartesian product) $R \times S$.
- Selection $\sigma_{n=a}(R)$ with a a constant.
- Arbitrary constant tables in queries (the constants in the lecture only had one single column and one single row; generalise this to any number of constants and rows)

Exercise 1.4. Consider the following identities and decide for each whether it is true or false. If true, prove your answer using the definitions from the lecture; if false, give a counterexample.

1. $R \bowtie S = S \bowtie R$
2. $R \bowtie (S \bowtie T) = (R \bowtie S) \bowtie T$.

Exercise 1.5. Suggest how to write the relational algebra operations for using the unnamed perspective. What changes?

Exercise 1.6. We have seen above that $\cap$ can be expressed in terms of the other standard operators of relational algebra. Indeed, the set of operations $\{\sigma, \pi, \cup, -, \bowtie, \delta\}$ can express all queries of relational algebra: it is complete. Try to show that it is not possible to reduce this set any further:

For the following operators, give an informal argument for why they are necessary: $\{\pi, \bowtie, \delta\}$. Give an idea for how to prove that we cannot do without the other operators.