

Complexity Theory
Exercise 7: Alternation
13th December 2023

Exercise 7.1. Describe a polynomial-time ATM solving **EXACT INDEPENDENT SET**:

Input: Given a graph G and some number k .

Question: Does there exist a maximal independent set in G of size exactly k ?

Find a level of the polynomial hierarchy where this problem is contained in.

Exercise 7.2. Consider the Japanese game *go-moku* that is played by two players X and O on a 19×19 board. Players alternately place markers on the board, and the first one to have five of its markers in a row, column, or diagonal wins.

Consider the generalised version of go-moku on an $n \times n$ board. Say that a *position* of go-moku is a placement of markers on such a board as it could occur during the game. Define

$$\mathbf{GM} = \{ \langle B \rangle \mid B \text{ is a position of go-moku where } X \text{ has a winning strategy} \}.$$

Describe a polynomial-time ATM solving **GM** and informally argue why this problem is not in any level of the polynomial hierarchy.

Exercise 7.3. Show that $\mathbf{AEXP TIME} = \mathbf{EXPSPACE}$.

Exercise 7.4. Show that $\Sigma_2\mathbf{QBF}$ is complete for $\Sigma_2\mathbf{P}$.

Exercise 7.5. Show that if $\mathbf{P} = \mathbf{NP}$, then $\mathbf{P} = \mathbf{PH}$.