Complexity Theory
Exercise 4: Time Complexity

Exercise 4.1. Show that P is closed under concatenation and star.

Exercise 4.2. Consider the problem **CLIQUE**:

Input: An undirected graph G and some $k \in \mathbb{N}$
Question: Does there exists a clique in G of size at least k?

Suppose **CLIQUE** can be solved in time $T(n)$ for some $T \colon \mathbb{N} \to \mathbb{N}$ with $T(n) \geq n$ for all $n \in \mathbb{N}$. Furthermore, show that then the optimisation problem

Input: An undirected graph G
Compute: A clique in G of maximal size

can be computed in time $\mathcal{O}(n \cdot T(n))$. You can assume that T is monotone.

Exercise 4.3. Show that if a language L is NP-complete, then $\overline{L}$ is coNP-complete.

Exercise 4.4. Show that if $P = NP$, then a polynomial-time algorithm exists that produces a satisfying assignment of a given satisfiable propositional formula.

Exercise 4.5. Show that finding paths of a given length in undirected graphs, i.e.,

$$\textbf{Path} = \{ \langle G, s, t, k \rangle \mid G \text{ contains a simple path from } s \text{ to } t \text{ of length } k \}$$

is NP-complete.

∗ **Exercise 4.6.** Let $A \subseteq 1^*$. Show that if A is NP-complete, then $P = NP$.
    Proceed as follows: Consider a polynomial-time reduction f from SAT to A. For a formula φ, let φ_{0100} be the reduced formula where variables x_1, x_2, x_3, x_4 in φ are set to the values 0, 1, 0, 0, respectively. (The particular choice of 4 variables as well as of 0100 is arbitrary here) What happens when one applies f to all of these exponentially many reduced formulas?