# SAT-Based Approaches to Reasoning about Argumentation Frameworks*

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Argumentation is a central topic in modern Artificial Intelligence (AI) research [Bench-Capon and Dunne, 2007], providing interesting research questions to researchers with different backgrounds, from computational complexity theory and automated reasoning to philosophy and social sciences, not forgetting application-oriented work in domains such as legal reasoning, multi-agent systems, and decision support. Argumentation frameworks (AFs) [Dung, 1995] have become the graph-based formal model of choice for many approaches to argumentation in AI, with semantics defining sets of jointly acceptable arguments, i.e., extensions.

System implementations for reasoning over AFs have recently received notable attention. Many of the central AF reasoning tasks can be represented in a natural way as Boolean combinations of logical constraints via developing propositional (Boolean satisfiability, SAT) encodings. This is true from both what we here refer to as *static* (or *non-dynamic*) problems, as well as problems related to *AF dynamics*, dealing with adjusting (or revising) a given AF to support new knowledge represented as extensions the AF should support. Interestingly, the study of AF dynamics gives rise to optimization problems, inviting the employment of maximum satisfiability (MaxSAT) solvers, the optimization counterpart of SAT, relying again heavily on SAT solvers.

The state-of-the-art SAT solver technology readily available today offers the core NP decision engines employed in many of the current state-of-the-art argumentation reasoning systems [Thimm and Villata, 2015]. Notably, the use of SAT solvers is not restricted to problem domains in NP. Rather, SAT solvers allow for solving hard decision problems presumably well beyond NP via harnessing instantiations of the general SAT-based *counterexample-guided abstraction refinement* (CEGAR) approach. In short, SAT-based CEGAR is based on iterative and incremental applications of SAT solvers, iteratively solving a sequence of abstractions and ruling out non-solutions through counterexample-based refinements to the abstraction towards finding one or more solutions to the actual problem instance at hand. As complexity-theoretically very challenging problems are abundant in AF reasoning—various types of decision and optimization problems under different AF semantics exhibiting completeness for different levels of the polynomial hierarchy—developing CEGAR-type SAT-based procedures for AF reasoning tasks is an intuitive choice.

The development of SAT-based procedures for AF reasoning tasks poses interesting research challenges of both theoretical and more applied nature.

*Complexity-theoretic analysis.* Understanding the complexity of AF reasoning tasks with respect to different parameterizations (AF semantics, reasoning modes, and other problem-specific parameters) is essential for understanding whether a specific reasoning task allows for direct SAT encodings (in NP) and on the other hand is not "too trivial" for SAT solvers (NP-complete, or at least not solvable in close to linear time). For reasoning tasks complete for higher levels of the polynomial hierarchy ($\Sigma_i^p$ / $\Pi_i^p$ complete for some $i > 1$), the level $i$ on which a specific task is situated gives guidelines on the requirements for SAT-based CEGAR suitable for the task, connecting theory to practice.

*NP encodings and CEGAR.* Development of SAT-based approaches is thus guided by complexity analysis for choosing the "right" approach to the AF reasoning task at hand. For problems in NP, a challenge is to develop reasonably compact direct SAT encodings (for decision problems) or MaxSAT encodings (or other constraint optimization formulations, for optimization problems) for the problem. Compactness here refers to ensuring scalability to larger AFs (with the understanding that, at times, SAT solvers can readily solve instances with millions or even tens of millions of variables and clauses [Järvisalo *et al.*, 2012]). However, a great challenge often is to understand the fundamental interplay between SAT encodings and the internal search techniques applied in different solvers. For example, using *more* variables in an encoding, or including redundant clauses, can at times guide the solver to decide instances faster. For CEGAR, a suitable NP-abstraction is needed, as well as refinement strategies which effectively rule out non-solutions from consideration.

*Implementation-level details.* From encodings and procedures to implementation, the choice of the SAT and MaxSAT solvers can have a noticeable impact on scalability and efficiency, in connection to the interplay between the underlying structure of a specific AF reasoning problem, the SAT/MaxSAT encoding, and the search techniques and heuristics applying within the solvers. The incremental APIs offered by some of the central SAT solvers also play a key

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role in implementing CEGAR-style iterative approaches. The use of MaxSAT solvers in CEGAR has been less studied, and poses more challenges, e.g. in that few MaxSAT solvers offer APIs, and still only few are available in open source.

We have recently applied this combination of theory and practice successfully to different types of AF reasoning task of both static and dynamic nature, as shortly outlined next.

**Cegartix**: *SAT-based CEGAR procedures for acceptance problems under various semantics*. A successful approach to static AF reasoning is provided by our CEGARTIX system [Dvořák *et al.*, 2014]. Implementing a SAT-based CEGAR approach to second-level complete skeptical and credulous acceptance problems, the system ranked at the top on second-level problems in the 1st International Competition on Computational Models of Argumentation.

**Pakota**: *MaxSAT-based encodings and CEGAR procedures for extension enforcement and status enforcement*. Addressing the so-called extension enforcement problem [Baumann, 2012; Bisquert *et al.*, 2013; Coste-Marquis *et al.*, 2015] in abstract argumentation and its generalizations, in [Wallner *et al.*, 2016] we provide a nearly complete computational complexity map of fixed-argument extension enforcement under various major AF semantics, with results ranging from polynomial-time algorithms to completeness for the second-level of the polynomial hierarchy. Complementing the complexity results, we give algorithms for NP-hard extension enforcement via constrained optimization. Going beyond NP, we propose novel MaxSAT-based CEGAR for the second-level complete problems, as well as an open-source system implementation of the approach. As a continuation, we have generalized the approach to the so-called *status enforcement problem* [Niskanen *et al.*, 2016a], bringing together concepts from both static credulous/skeptical acceptance and AF dynamics, most closely, extension enforcement.

**AF synthesis**: *MaxSAT approaches to synthesizing AFs from examples*. A fundamental knowledge representational aspect related to AFs is *realizability* [Dunne *et al.*, 2015], i.e., the question of whether a specific AF semantics allows for *exactly* representing a given set of extensions as an AF [Dunne *et al.*, 2015; Baumann *et al.*, 2014; Dyrkolbotn, 2014; Pührer, 2015; Linsbichler *et al.*, 2016; 2015]. Realizability is quite strict in that a set $E$ of extensions is considered realizable (under a specific semantics) if and only if there is an AF the extensions of which are *exactly* those in $E$; this requires that we have *complete* knowledge of the extensions of interest, and, in order to actually construct a corresponding AF of interest, relies on the assumption that the set of extensions are *not conflicting* in terms of allowing them to be exactly represented by an AF. Recently in [Niskanen *et al.*, 2016b], we generalized the concept of realizability to accommodate incomplete and noise information on extensions, proposing what we call the *AF synthesis problem*, relaxing the notion of realizability to incomplete information noisy (weighted) settings. Establishing NP-complete and tractable cases of AF synthesis, we have developed a first MaxSAT-based approach to optimal AF synthesis, again going from complexity-theoretic analysis to on actual implemented system for AF synthesis.

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