

Introduction to Formal Concept Analysis

Exercise Sheet 3, Winter Semester 2017/18

Exercise 1 (computing and drawing formal concepts)

Given the formal context in Table 1,

1. compute all formal concepts of that context,
2. draw and label the line diagram of the concept lattice, and
3. check if the diagram is correct.

Can you discover implications in that context? If yes, which? Do these implications hold in general? If not, how could they be become invalid?

Tabelle 1: Grobian Gans: *Die Ducks. Psychogramm einer Sippe*. Rowohlt, Reinbek bei Hamburg 1972, ISBN 3-499-11481-X

	generation			sex		financial status		
	older	middle	younger	male	female	rich	carefree	indebted
Tick			×	×			×	
Trick			×	×			×	
Track			×	×			×	
Donald		×		×				×
Daisy		×			×		×	
Gustav		×		×			×	
Dagobert	×			×		×		
Annette	×				×		×	
Primus v. Quack	×			×			×	

Solution:

- 1) We use the intersection method to compute all formal concepts manually.
 1. Write the attribute extents to a list

No.	extent	found as
e1	:= {Dagobert, Annette, Primus v. Quack}	{older}'
e2	:= {Donald, Daisy, Gustav}	{middle}'
e3	:= {Tick, Trick, Track}	{younger}'
e4	:= {Tick, Trick, Track, Donald, Gustav, Dagobert, Primus v. Quack}	{male}'
e5	:= {Daisy, Annette}	{female}'
e6	:= {Dagobert}	{rich}'
e7	:= {Tick, Trick, Track, Daisy, Gustav, Annette, Primus v. Quack}	{carefree}'
e8	:= {Donald}	{indebted}'

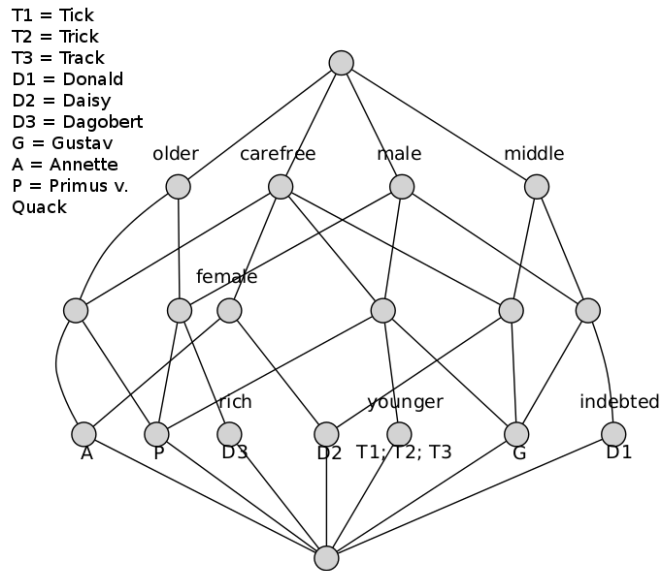
2. Compute all pairwise intersections of objects, and add G.

No.	extent	found as
e9	$:= \emptyset$	$e1 \cap e2$
e10	$:= \{\text{Dagobert, Primus v. Quack}\}$	$e1 \cap e4$
e11	$:= \{\text{Annette}\}$	$e1 \cap e5$
e12	$:= \{\text{Annette, Primus v. Quack}\}$	$e1 \cap e7$
e13	$:= \{\text{Donald, Gustav}\}$	$e2 \cap e4$
e14	$:= \{\text{Daisy}\}$	$e2 \cap e5$
e15	$:= \{\text{Daisy, Gustav}\}$	$e2 \cap e7$
e16	$:= \{\text{Tick, Trick, Track, Gustav, Primus v. Quack}\}$	$e4 \cap e7$
e17	$:= \{\text{Primus v. Quack}\}$	$e7 \cap e10$
e18	$:= \{\text{Gustav}\}$	$e7 \cap e13$
e19	$:= \{\text{Tick, Trick, Track, Donald, Daisy, Gustav, Dagobert, Annette, Primus v. Quack}\}$	Add G

3. Compute the intents of the objects.

No.	(extents , intents)
e1	({Dagobert, Annette, Primus v. Quack} , {older})
e2	({Donald, Daisy, Gustav} , {middle})
e3	({Tick, Trick, Track} , {younger, male, carefree})
e4	({Tick, Trick, Track, Donald, Gustav, Dagobert, Primus v. Quack} , {male})
e5	({Daisy, Annette} , {female, carefree})
e6	({Dagobert} , {older, male, rich})
e7	({Tick, Trick, Track, Daisy, Gustav, Annette, Primus v. Quack} , {carefree})
e8	({Donald} , {middle, male, indebted})
e9	(\emptyset , M = set of all attributes)
e10	({Dagobert, Primus v. Quack} , {older, male})
e11	({Annette} , {older, female, carefree})
e12	({Annette, Primus v. Quack} , {older, carefree})
e13	({Donald, Gustav} , {middle, male})
e14	({Daisy} , {middle, female, carefree})
e15	({Daisy, Gustav} , {middle, carefree})
e16	({Tick, Trick, Track, Gustav, Primus v. Quack} , {male, carefree})
e17	({Primus v. Quack} , {older, male, carefree})
e18	({Gustav} , {middle, male, carefree})
e19	(G = set of all objects , \emptyset)

2) Drawing and labelling line diagram of the concept lattice.



Exercise 2 (clarified and reduced contexts)

Consider the first formal context from the lecture, depicted in Table 2.

1. Provide a set of objects and a set of attributes which, when removed from the context, yield a clarified context whose concept lattice is isomorphic to that of the original context.
2. Considering the clarified context thus obtained, determine the reducible objects and attributes.

Solution:

1. Removing the following objects leads to a clarified context:

- Pinnacles Natl. Mon.
- Point Reyes Natl. Seashore
- Redwood Natl. Park
- Sequoia Natl. Park
- Wiskeytown-Shasta-Trinity Natl. Recr. Area

No attributes need to be removed.

2. The reducible objects of the remaining context are

- Devils Postpile Natl. Mon.
- John Muir Natl. Historic Site
- Joshua Tree Natl. Mon.
- Yosemite Natl. Park

There are no reducible attributes.

Tabelle 2: National Parks in California

National Parks in California	<i>NPS Guided Tours</i>	<i>Hiking</i>	<i>Horseback Riding</i>	<i>Swimming</i>	<i>Boating</i>	<i>Fishing</i>	<i>Bicycle Trail</i>	<i>Cross Country Trail</i>
Cabrillo Natl. Mon.						×	×	
Channel Islands Natl. Park		×		×		×		
Death Valley Natl. Mon.	×	×	×	×			×	
Devils Postpile Natl. Mon.	×	×	×	×		×		
Fort Point Natl. Historic Site	×					×		
Golden Gate Natl. Recreation Area	×	×	×	×		×	×	
John Muir Natl. Historic Site	×							
Joshua Tree Natl. Mon.	×	×	×					
Kings Canyon Natl. Park	×	×	×			×		×
Lassen Volcanic Natl. Park	×	×	×	×	×	×		×
Lava Beds Natl. Mon.	×	×						
Muir Woods Natl. Mon.		×						
Pinnacles Natl. Mon.		×						
Point Reyes Natl. Seashore	×	×	×	×		×	×	
Redwood Natl. Park	×	×	×	×		×		
Santa Monica Mts. Natl. Recr. Area	×	×	×	×	×	×		
Sequoia Natl. Park	×	×	×			×		×
Whiskeytown-Shasta-Trinity Natl. Recr. Area	×	×	×	×	×	×		
Yosemite Natl. Park	×	×	×	×	×	×	×	×