Exploring Faulty Data

Daniel Borchmann

TU Dresden

2015-06-26

http://www.sealifeconservation.org.au/adopt-an-animal/

The platypus is a counterexample to

$$\{\,\text{Venomous}, \text{Mammal}\,\} \to \bot.$$

The platypus is a counterexample to

$$\{\,\mathrm{Venomous}, \mathrm{Mammal}\,\} \to \bot.$$

Accept this Counterexample?

The platypus is a counterexample to

$$\{\,\mathrm{Venomous}, \mathrm{Mammal}\,\} \rightarrow \bot.$$

Accept this Counterexample?

It depends

The platypus is a counterexample to

$$\{\,\text{Venomous}, \text{Mammal}\,\} \to \bot.$$

Accept this Counterexample?

It depends

- True if all mammals are considered

The platypus is a counterexample to

$$\{\,\text{Venomous}, \text{Mammal}\,\} \to \bot.$$

Accept this Counterexample?

It depends

- True if all mammals are considered
- But exceptional, i.e., this counterexample is *rare*

The platypus is a counterexample to

$$\{ \text{Venomous}, \text{Mammal} \} \to \bot.$$

Accept this Counterexample?

It depends

- True if all mammals are considered
- But exceptional, i.e., this counterexample is *rare*
- In general not desired (handle this special case elsewhere)

The platypus is a counterexample to

$$\{\,\text{Venomous}, \text{Mammal}\,\} \to \bot.$$

Accept this Counterexample?

It depends

- True if all mammals are considered
- But exceptional, i.e., this counterexample is *rare*
- In general not desired (handle this special case elsewhere)

How to decide?

The platypus is a counterexample to

$$\{\,\text{Venomous}, \text{Mammal}\,\} \rightarrow \bot.$$

Accept this Counterexample?

It depends

- True if all mammals are considered
- But exceptional, i.e., this counterexample is *rare*
- In general not desired (handle this special case elsewhere)

How to decide?

External source of information needed!

Attribute Exploration

Expert

Algorithm

$\mathbb{K}$	m_1	$\ldots$	m_n
g_1		$\ldots$	
$\vdots$			
g_k		$\ldots$	

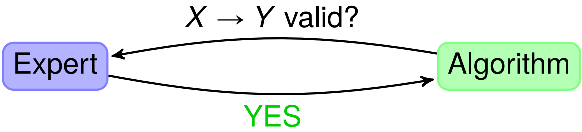
$$\mathcal{S} = \{ A_1 \to B_1, \ldots, A_\ell \to B_\ell \}$$

Attribute Exploration

$\mathbb{K}$	m_1	$\ldots$	m_n
g_1		$\ldots$	
$\vdots$			
g_k		$\ldots$	

$$\mathcal{S} = \{ A_1 \to B_1, \ \ldots \ A_\ell \to B_\ell \}$$

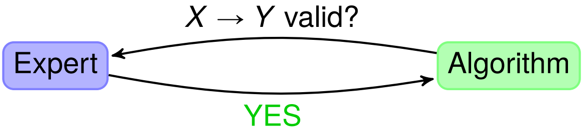
Attribute Exploration



$\mathbb{K}$	m_1	$\ldots$	m_n
g_1		$\ldots$	
$\vdots$			
g_k		$\ldots$	

$$\mathcal{S} = \{ A_1 \to B_1, \ldots, A_\ell \to B_\ell \}$$

Attribute Exploration



$\mathbb{K}$	m_1	$\ldots$	m_n
g_1		$\ldots$	
$\vdots$			
g_k		$\ldots$	

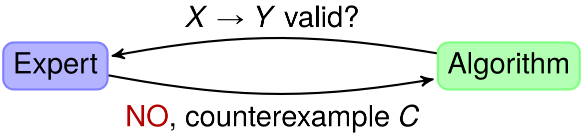
$$\mathcal{S} = \{ A_1 \to B_1, \ldots, A_\ell \to B_\ell, X \to Y \}$$

Attribute Exploration

$\mathbb{K}$	m_1	$\ldots$	m_n
g_1		$\ldots$	
$\vdots$			
g_k		$\ldots$	

$$\mathcal{S} = \{ A_1 \to B_1, \ldots A_\ell \to B_\ell \}$$

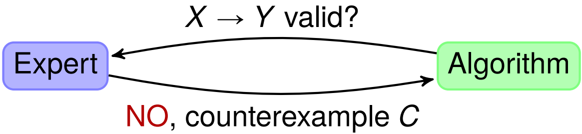
Attribute Exploration



$\mathbb{K}$	m_1	$\dots$	m_n
g_1		$\dots$	
$\vdots$			
g_k		$\dots$	

$$\mathcal{S} = \{ A_1 \to B_1, \ \dots \ A_\ell \to B_\ell \}$$

Attribute Exploration



$\mathbb{K}$	m_1	$\dots$	m_n
g_1		$\dots$	
$\vdots$			
g_k		$\dots$	
g_{k+1}	——	C	——

$$\mathcal{S} = \{ A_1 \to B_1, \quad \dots \quad A_\ell \to B_\ell \}$$

Attribute Exploration

Expert

Algorithm

$\mathbb{K}$	m_1	$\dots$	m_n
g_1		$\dots$	
$\vdots$			
g_k		$\dots$	

$$\mathcal{S} = \{ A_1 \to B_1, \dots A_\ell \to B_\ell \}$$

Attribute Exploration

Expert

Algorithm

$\mathbb{K}$	m_1	$\dots$	m_n
g_1		$\dots$	
$\vdots$			
g_k		$\dots$	

$$\mathcal{S} = \{\, A_1 \to B_1, \quad \dots \quad A_\ell \to B_\ell \,\}$$

- Upon termination, $\mathrm{Cn}(\mathcal{S}) = \mathrm{Th}(\mathbb{K})$ and $\mathcal{S}$ is a base of the implicational knowledge of the expert

Attribute Exploration

Expert

Algorithm

$\mathbb{K}$	m_1	$\dots$	m_n
g_1		$\dots$	
$\vdots$			
g_k		$\dots$	

$$\mathcal{S} = \{\, A_1 \to B_1,\ \dots\ A_\ell \to B_\ell \,\}$$

- Upon termination, $\mathrm{Cn}(\mathcal{S}) = \mathrm{Th}(\mathbb{K})$ and $\mathcal{S}$ is a base of the implicational knowledge of the expert
- Implications asked are always valid

Attribute Exploration

Expert

Algorithm

$\mathbb{K}$	m_1	$\dots$	m_n
g_1		$\dots$	
$\vdots$			
g_k		$\dots$	

$$\mathcal{S} = \{ A_1 \to B_1, \dots A_\ell \to B_\ell \}$$

- Upon termination, $\text{Cn}(\mathcal{S}) = \text{Th}(\mathbb{K})$ and $\mathcal{S}$ is a base of the implicational knowledge of the expert
- Implications asked are always valid

Problem

Data may contain errors or unwanted special cases

Attribute Exploration

Expert

Algorithm

$\mathbb{K}$	m_1	$\dots$	m_n
g_1		$\dots$	
$\vdots$			
g_k		$\dots$	

$$\mathcal{S} = \{ A_1 \to B_1, \dots, A_\ell \to B_\ell \}$$

- Upon termination, $\mathrm{Cn}(\mathcal{S}) = \mathrm{Th}(\mathbb{K})$ and $\mathcal{S}$ is a base of the implicational knowledge of the expert
- Implications asked are always valid

Problem

Data may contain errors or unwanted special cases $\rightsquigarrow$ ask implications with *high confidence*

Definition

Define

$$\text{conf}_{\mathbb{K}}(X \rightarrow Y) := \begin{cases} 1 & X' = \varnothing \\ \frac{|(X \cup Y)'|}{|X'|} & \text{otherwise} \end{cases}$$

Definition

Define

$$\mathsf{conf}_{\mathbb{K}}(X \to Y) := \begin{cases} 1 & X' = \varnothing \\ \frac{|(X \cup Y)'|}{|X'|} & \text{otherwise} \end{cases}$$

For $c \in [0, 1]$ set

$$\mathsf{Th}_c(\mathbb{K}) := \{\, X \to Y \mid \mathsf{conf}_{\mathbb{K}}(X \to Y) \geqslant c \,\}.$$

Definition

Define

$$\mathrm{conf}_{\mathbb{K}}(X \to Y) := \begin{cases} 1 & X' = \varnothing \\ \frac{|(X \cup Y)'|}{|X'|} & \text{otherwise} \end{cases}$$

For $c \in [0, 1]$ set

$$\mathrm{Th}_c(\mathbb{K}) := \{\, X \to Y \mid \mathrm{conf}_{\mathbb{K}}(X \to Y) \geqslant c \,\}.$$

Idea

Extend exploration such that implications with *high confidence* are asked.

Definition

Let M be a finite set. Then an *expert* p on M is a mapping

$$p\colon \operatorname{Imp}(M) \to \mathfrak{P}(M) \cup \{\top\}$$

such that

Definition

Let M be a finite set. Then an *expert* p on M is a mapping

$$p\colon \operatorname{Imp}(M) \to \mathfrak{P}(M) \cup \{\top\}$$

such that

- $p(X \to Y) = C \neq \top$ implies $X \subseteq C$ and $Y \nsubseteq C$,

Definition

Let M be a finite set. Then an *expert* p on M is a mapping

$$p\colon \operatorname{Imp}(M) \to \mathfrak{P}(M) \cup \{\top\}$$

such that

- $p(X \to Y) = C \neq \top$ implies $X \subseteq C$ and $Y \nsubseteq C$,
  (*expert gives counterexamples to false implications*)

Definition

Let M be a finite set. Then an *expert* p on M is a mapping

$$p\colon \operatorname{Imp}(M) \to \mathfrak{P}(M) \cup \{\top\}$$

such that

- $p(X \to Y) = C \neq \top$ implies $X \subseteq C$ and $Y \nsubseteq C$,
  (*expert gives counterexamples to false implications*)
- $p(U \to V) = \top, p(X \to Y) = C \neq \top$ implies $U \nsubseteq C$ or $V \subseteq C$.

Definition

Let M be a finite set. Then an *expert* p on M is a mapping

$$p\colon \operatorname{Imp}(M) \to \mathfrak{P}(M) \cup \{\top\}$$

such that

- $p(X \to Y) = C \neq \top$ implies $X \subseteq C$ and $Y \nsubseteq C$,
  (*expert gives counterexamples to false implications*)
- $p(U \to V) = \top, p(X \to Y) = C \neq \top$ implies $U \nsubseteq C$ or $V \subseteq C$.
  (*counterexamples do not invalidate correct implications*)

Definition

To *explore* $\mathrm{Th}_c(\mathbb{K})$ *with expert p with background knowledge* $\mathcal{S}$ means to find a base of

$$\mathrm{Th}(p) \cap \mathrm{Th}_c(\mathbb{K})$$

with background knowledge $\mathcal{S}$,

Definition

To *explore* $\text{Th}_c(\mathbb{K})$ *with expert p with background knowledge* $\mathcal{S}$ means to find a base of

$$\text{Th}(p) \cap \text{Th}_c(\mathbb{K})$$

with background knowledge $\mathcal{S}$, i.e., to compute a set $\mathcal{B} \subseteq \text{Imp}(M)$ such that

$$\text{Cn}(\mathcal{B} \cup \mathcal{S}) = \text{Cn}\big(\text{Th}(p) \cap \text{Th}_c(\mathbb{K})\big).$$

Classical Attribute Exploration

	a	b	c
1	$\times$	.	.

Classical Attribute Exploration

	a	b	c
1	×	.	.

$\emptyset$

$\{a\}$

$\{b\}$

$\{a, b\}$

$\{c\}$

$\{a, c\}$

$\{b, c\}$

Classical Attribute Exploration

	a	b	c
1	$\times$	.	.

$$\varnothing \to \varnothing''$$
$$\{\, a \,\} \to \{\, a \,\}''$$
$$\{\, b \,\} \to \{\, b \,\}''$$
$$\{\, a, b \,\} \to \{\, a, b \,\}''$$
$$\{\, c \,\} \to \{\, c \,\}''$$
$$\{\, a, c \,\} \to \{\, a, c \,\}''$$
$$\{\, b, c \,\} \to \{\, b, c \,\}''$$

Classical Attribute Exploration

	a	b	c
1	$\times$	.	.

$$\begin{aligned}
\emptyset &\to \emptyset'' & = \{a\} \\
\{a\} &\to \{a\}'' \\
\{b\} &\to \{b\}'' \\
\{a,b\} &\to \{a,b\}'' \\
\{c\} &\to \{c\}'' \\
\{a,c\} &\to \{a,c\}'' \\
\{b,c\} &\to \{b,c\}''
\end{aligned}$$

Classical Attribute Exploration

	a	b	c
1	$\times$	.	.

$$\varnothing \to \varnothing'' \qquad = \{a\} \checkmark$$
$$\{a\} \to \{a\}''$$
$$\{b\} \to \{b\}''$$
$$\{a,b\} \to \{a,b\}''$$
$$\{c\} \to \{c\}''$$
$$\{a,c\} \to \{a,c\}''$$
$$\{b,c\} \to \{b,c\}''$$

Classical Attribute Exploration

	a	b	c
1	$\times$	.	.

$$\varnothing \to \varnothing'' \quad = \{a\} \checkmark$$
$$\{a\} \to \{a\}''$$
$$\{b\} \to \{b\}''$$
$$\{a,b\} \to \{a,b\}''$$
$$\{c\} \to \{c\}''$$
$$\{a,c\} \to \{a,c\}''$$
$$\{b,c\} \to \{b,c\}''$$

Classical Attribute Exploration

	a	b	c
1	×	.	.

$$\varnothing \to \varnothing'' = \{a\} \checkmark$$

$$\{a\} \to \{a\}'' = \{a\}$$

$$\sout{\{b\} \to \{b\}''}$$

$$\{a,b\} \to \{a,b\}''$$

$$\sout{\{c\} \to \{c\}''}$$

$$\{a,c\} \to \{a,c\}''$$

$$\sout{\{b,c\} \to \{b,c\}''}$$

Classical Attribute Exploration

	a	b	c
1	$\times$	.	.

$$\emptyset \to \emptyset'' = \{a\} \checkmark$$

$$\{a\} \to \{a\}'' = \{a\}$$

~~$\{b\} \to \{b\}''$~~

$$\{a, b\} \to \{a, b\}'' = \{a, b, c\}$$

~~$\{c\} \to \{c\}''$~~

$$\{a, c\} \to \{a, c\}''$$

~~$\{b, c\} \to \{b, c\}''$~~

Classical Attribute Exploration

	a	b	c
1	$\times$	.	.

$$\emptyset \to \emptyset'' = \{a\} \checkmark$$

$$\{a\} \to \{a\}'' = \{a\}$$

~~$\{b\} \to \{b\}''$~~

$$\{a,b\} \to \{a,b\}'' = \{a,b,c\} ✗$$

~~$\{c\} \to \{c\}''$~~

$$\{a,c\} \to \{a,c\}''$$

~~$\{b,c\} \to \{b,c\}''$~~

Classical Attribute Exploration

	a	b	c
1	$\times$	.	.
2	$\times$	$\times$	.

$$\emptyset \to \emptyset'' = \{a\} \checkmark$$

$$\{a\} \to \{a\}'' = \{a\}$$

$$\sout{\{b\} \to \{b\}''}$$

$$\{a,b\} \to \{a,b\}'' = \{a,b,c\} ✗$$

$$\sout{\{c\} \to \{c\}''}$$

$$\{a,c\} \to \{a,c\}''$$

$$\sout{\{b,c\} \to \{b,c\}''}$$

Classical Attribute Exploration

	a	b	c
1	×	.	.
2	×	×	.

$$\varnothing \to \varnothing'' = \{a\} \checkmark$$
$$\{a\} \to \{a\}'' = \{a\}$$
$$\sout{\{b\} \to \{b\}''}$$
$$\{a,b\} \to \{a,b\}'' = \{a,b\}$$
$$\sout{\{c\} \to \{c\}''}$$
$$\{a,c\} \to \{a,c\}''$$
$$\sout{\{b,c\} \to \{b,c\}''}$$

Classical Attribute Exploration

	a	b	c
1	$\times$	.	.
2	$\times$	$\times$	.

$$\emptyset \to \emptyset'' = \{a\} \checkmark$$
$$\{a\} \to \{a\}'' = \{a\}$$
$$\cancel{\{b\} \to \{b\}''}$$
$$\{a,b\} \to \{a,b\}'' = \{a,b\}$$
$$\cancel{\{c\} \to \{c\}''}$$
$$\{a,c\} \to \{a,c\}'' = \{a,b,c\}$$
$$\cancel{\{b,c\} \to \{b,c\}''}$$

Classical Attribute Exploration

	a	b	c
1	×	.	.
2	×	×	.

$$\emptyset \to \emptyset'' = \{a\} \checkmark$$

$$\{a\} \to \{a\}'' = \{a\}$$

~~$\{b\} \to \{b\}''$~~

$$\{a,b\} \to \{a,b\}'' = \{a,b\}$$

~~$\{c\} \to \{c\}''$~~

$$\{a,c\} \to \{a,c\}'' = \{a,b,c\} \checkmark$$

~~$\{b,c\} \to \{b,c\}''$~~

Classical Attribute Exploration

	a	b	c
1	×	.	.
2	×	×	.

$$\varnothing \to \varnothing'' = \{a\} \checkmark$$
$$\{a\} \to \{a\}'' = \{a\}$$
$$\sout{\{b\} \to \{b\}''}$$
$$\{a,b\} \to \{a,b\}'' = \{a,b\}$$
$$\sout{\{c\} \to \{c\}''}$$
$$\{a,c\} \to \{a,c\}'' = \{a,b,c\} \checkmark$$
$$\sout{\{b,c\} \to \{b,c\}''}$$

Note

Classical Attribute Exploration

	a	b	c
1	×	.	.
2	×	×	.

$$\varnothing \to \varnothing'' = \{a\} \checkmark$$

$$\{a\} \to \{a\}'' = \{a\}$$

$$\sout{\{b\} \to \{b\}''}$$

$$\{a,b\} \to \{a,b\}'' = \{a,b\}$$

$$\sout{\{c\} \to \{c\}''}$$

$$\{a,c\} \to \{a,c\}'' = \{a,b,c\} \checkmark$$

$$\sout{\{b,c\} \to \{b,c\}''}$$

Note

- Order of premises extends $\subseteq$

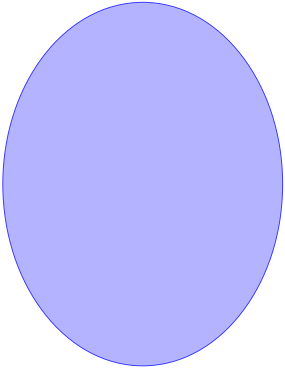
Classical Attribute Exploration

	a	b	c
1	$\times$	.	.
2	$\times$	$\times$	.

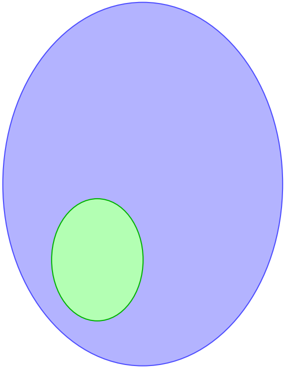
$$\emptyset \to \emptyset'' = \{a\} \checkmark$$
$$\{a\} \to \{a\}'' = \{a\}$$
$$\require{cancel}\cancel{\{b\} \to \{b\}''}$$
$$\{a,b\} \to \{a,b\}'' = \{a,b\}$$
$$\cancel{\{c\} \to \{c\}''}$$
$$\{a,c\} \to \{a,c\}'' = \{a,b,c\} \checkmark$$
$$\cancel{\{b,c\} \to \{b,c\}''}$$

Note

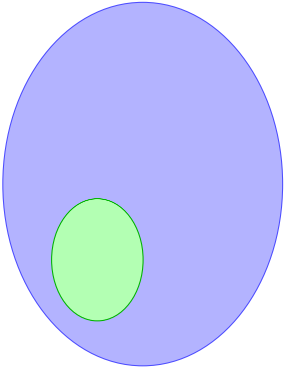
- Order of premises extends $\subseteq$
- Premises are always closed under known implications



- *Interesting Implications* $\mathcal{L}$



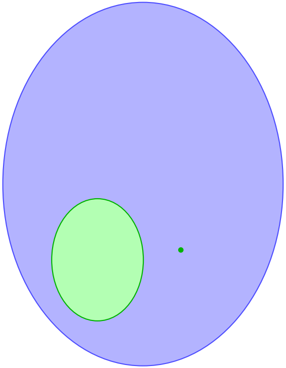
- *Interesting Implications* $\mathcal{L}$
- *Known Implications* $\mathcal{S}$



- *Interesting Implications* $\mathcal{L}$
- *Known Implications* $\mathcal{S}$

Question

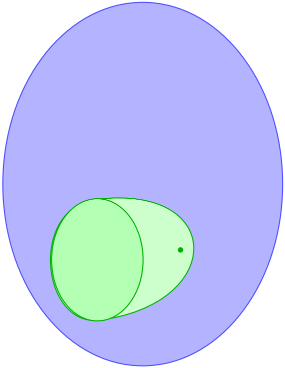
Which interesting but unknown implications are *valid* in our domain?



- *Interesting Implications* $\mathcal{L}$
- *Known Implications* $\mathcal{S}$
- Expert can *confirm* implications,

Question

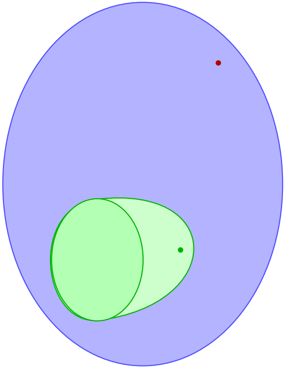
Which interesting but unknown implications are *valid* in our domain?



- *Interesting Implications* $\mathcal{L}$
- *Known Implications* $\mathcal{S}$
- Expert can *confirm* implications, extending $\mathcal{S}$

Question

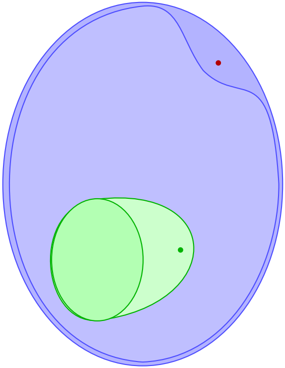
Which interesting but unknown implications are *valid* in our domain?



- *Interesting Implications* $\mathcal{L}$
- *Known Implications* $\mathcal{S}$
- Expert can *confirm* implications, extending $\mathcal{S}$
- Expert can *reject* implications, providing counterexamples,

Question

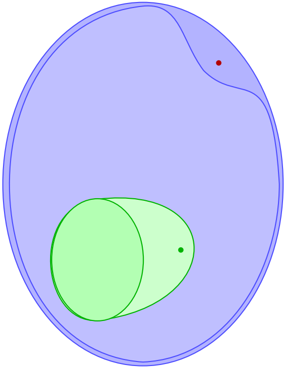
Which interesting but unknown implications are *valid* in our domain?



- *Interesting Implications* $\mathcal{L}$
- *Known Implications* $\mathcal{S}$
- Expert can *confirm* implications, extending $\mathcal{S}$
- Expert can *reject* implications, providing counterexamples, shrinking $\mathcal{L}$

Question

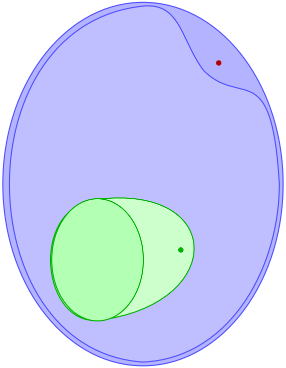
Which interesting but unknown implications are *valid* in our domain?



- *Interesting Implications* $\mathcal{L}$
- *Known Implications* $\mathcal{S}$
- Expert can *confirm* implications, extending $\mathcal{S}$
- Expert can *reject* implications, providing counterexamples, shrinking $\mathcal{L}$
- Iterate until $\mathcal{L} = \mathcal{S}$.

Question

Which interesting but unknown implications are *valid* in our domain?

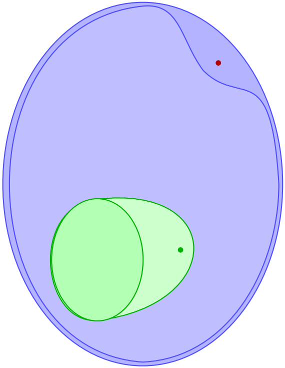


- *Interesting Implications* $\mathcal{L}$
- *Known Implications* $\mathcal{S}$
- Expert can *confirm* implications, extending $\mathcal{S}$
- Expert can *reject* implications, providing counterexamples, shrinking $\mathcal{L}$
- Iterate until $\mathcal{L} = \mathcal{S}$.

Question

Which interesting but unknown implications are *valid* in our domain?

- Classical Attribute Exploration: $\mathcal{L} = \mathrm{Th}(\mathbb{K})$



- *Interesting Implications* $\mathcal{L}$
- *Known Implications* $\mathcal{S}$
- Expert can *confirm* implications, extending $\mathcal{S}$
- Expert can *reject* implications, providing counterexamples, shrinking $\mathcal{L}$
- Iterate until $\mathcal{L} = \mathcal{S}$.

Question

Which interesting but unknown implications are *valid* in our domain?

- Classical Attribute Exploration: $\mathcal{L} = \mathrm{Th}(\mathbb{K})$
- Exploration by Confidence: $\mathcal{L} = \mathrm{Th}_c(\mathbb{K})$

First Idea

First Idea

Replace occurrence of $(\cdot)''$ with $\mathsf{Th}_c(\mathbb{K})(\cdot)$.

First Idea

Replace occurrence of $(\cdot)''$ with $\mathsf{Th}_c(\mathbb{K})(\cdot)$.

Problems

First Idea

Replace occurrence of $(\cdot)''$ with $\mathrm{Th}_c(\mathbb{K})(\cdot)$.

Problems

- It doesn't work,

First Idea

Replace occurrence of $(\cdot)''$ with $\mathrm{Th}_c(\mathbb{K})(\cdot)$.

Problems

- It doesn't work, i.e., exploration is only "approximative":

First Idea

Replace occurrence of $(\cdot)''$ with $\mathrm{Th}_c(\mathbb{K})(\cdot)$.

Problems

- It doesn't work, i.e., exploration is only "approximative":

$$\mathrm{Th}(p) \cap \mathrm{Cn}\big(\mathrm{Th}_c(\mathbb{K})\big) \supseteq \mathrm{Cn}(\mathcal{B} \cup \mathcal{S}) \supseteq \mathrm{Cn}\big(\mathrm{Th}(p) \cap \mathrm{Th}_c(\mathbb{K})\big)$$

First Idea

Replace occurrence of $(\cdot)''$ with $\mathsf{Th}_c(\mathbb{K})(\cdot)$.

Problems

- It doesn't work, i.e., exploration is only "approximative":

$$\mathsf{Th}(p) \cap \mathsf{Cn}\big(\mathsf{Th}_c(\mathbb{K})\big) \supseteq \mathsf{Cn}(\mathcal{B} \cup \mathcal{S}) \supseteq \mathsf{Cn}\big(\mathsf{Th}(p) \cap \mathsf{Th}_c(\mathbb{K})\big)$$

- Closures under $\mathsf{Th}_c(\mathbb{K})$ are (potentially) expensive

Idea

Idea

Instead of

$$X \to \mathrm{Th}_c(\mathbb{K})(X),$$

ask implications of the form

$$X \to \{\, m \in M \mid \mathrm{conf}_{\mathbb{K}}(X \to \{\, m \,\}) \geqslant c \,\}.$$

Idea

Instead of

$$X \to \mathrm{Th}_c(\mathbb{K})(X),$$

ask implications of the form

$$X \to \{\, m \in M \mid \mathrm{conf}_{\mathbb{K}}(X \to \{\, m \,\}) \geqslant c \,\}.$$

Problem

Doesn’t work either.

Choose

- $c := \frac{1}{2}$
- $\mathcal{S} := \{\, \{\, \mathrm{a} \,\} \to \{\, \mathrm{b} \,\} \,\}$
- p constantly $\top$

	a	b	c
1	×	×	.
2	×	×	.
3	×	×	.
4	×	×	×
5	×	×	×
6	×	.	×
7	×	.	×
8	.	.	.
9	.	.	.
10	.	.	.

Choose

- $c := \frac{1}{2}$
- $\mathcal{S} := \{\, \{\, \mathsf{a} \,\} \to \{\, \mathsf{b} \,\} \,\}$
- p constantly $\top$

Then

$$\mathrm{conf}_{\mathbb{K}}(\{\, \mathsf{a} \,\} \to \{\, \mathsf{c} \,\}) \geqslant c,$$

	a	b	c
1	×	×	.
2	×	×	.
3	×	×	.
4	×	×	×
5	×	×	×
6	×	.	×
7	×	.	×
8	.	.	.
9	.	.	.
10	.	.	.

Choose

- $c := \frac{1}{2}$
- $\mathcal{S} := \{ \{ \mathsf{a} \} \to \{ \mathsf{b} \} \}$
- p constantly $\top$

Then

$$\mathrm{conf}_{\mathbb{K}}(\{ \mathsf{a} \} \to \{ \mathsf{c} \}) \geqslant c,$$

but $\{ \mathsf{a} \}$ is not closed under $\mathcal{S}$,

	a	b	c
1	×	×	.
2	×	×	.
3	×	×	.
4	×	×	×
5	×	×	×
6	×	.	×
7	×	.	×
8	.	.	.
9	.	.	.
10	.	.	.

Choose

- $c := \frac{1}{2}$
- $\mathcal{S} := \{ \{ \mathsf{a} \} \to \{ \mathsf{b} \} \}$
- p constantly $\top$

Then

$$\mathrm{conf}_{\mathbb{K}}(\{ \mathsf{a} \} \to \{ \mathsf{c} \}) \geqslant c,$$

but $\{ \mathsf{a} \}$ is not closed under $\mathcal{S}$, and

$$\mathrm{conf}_{\mathbb{K}}(\{ \mathsf{b} \} \to \{ \mathsf{c} \}) = \frac{2}{5} < c,$$

$$\mathrm{conf}_{\mathbb{K}}(\{ \mathsf{a}, \mathsf{b} \} \to \{ \mathsf{c} \}) = \frac{2}{5} < c,$$

$$\mathrm{conf}_{\mathbb{K}}(\varnothing \to \{ \mathsf{c} \}) = \frac{4}{10} < c.$$

	a	b	c
1	×	×	.
2	×	×	.
3	×	×	.
4	×	×	×
5	×	×	×
6	×	.	×
7	×	.	×
8	.	.	.
9	.	.	.
10	.	.	.

Solution

Solution

Ensure: when

$$X \to \{\, m \in M \mid \operatorname{conf}_{\mathbb{K}}(X \to \{\, m \,\}) \geqslant c \,\}$$

is asked in iteration i,

Solution

Ensure: when

$$X \to \{\, m \in M \mid \mathrm{conf}_{\mathbb{K}}(X \to \{\, m \,\}) \geqslant c \,\}$$

is asked in iteration i, then for $Y'' \subsetneq X$

$$\mathrm{conf}_{\mathbb{K}}(Y'' \to \{\, n \,\}) \geqslant c \implies \mathcal{B}_i \cup \mathcal{S} \models (Y'' \to \{\, n \,\}).$$

Solution

Ensure: when

$$X \to \{\, m \in M \mid \mathrm{conf}_{\mathbb{K}}(X \to \{\, m \,\}) \geqslant c \,\}$$

is asked in iteration i, then for $Y'' \subsetneq X$

$$\mathrm{conf}_{\mathbb{K}}(Y'' \to \{\, n \,\}) \geqslant c \implies \mathcal{B}_i \cup \mathcal{S} \models (Y'' \to \{\, n \,\}).$$

Types of Implications asked

Solution

Ensure: when

$$X \to \{\, m \in M \mid \mathrm{conf}_{\mathbb{K}}(X \to \{\, m \,\}) \geqslant c \,\}$$

is asked in iteration i, then for $Y'' \subsetneq X$

$$\mathrm{conf}_{\mathbb{K}}(Y'' \to \{\, n \,\}) \geqslant c \implies \mathcal{B}_i \cup \mathcal{S} \models (Y'' \to \{\, n \,\}).$$

Types of Implications asked

- $X \to X''$, where X is closed under $\mathcal{B}_i \cup \mathcal{S}$, but not an intent

Solution

Ensure: when

$$X \to \{\, m \in M \mid \mathrm{conf}_{\mathbb{K}}(X \to \{\, m \,\}) \geqslant c \,\}$$

is asked in iteration i, then for $Y'' \subsetneq X$

$$\mathrm{conf}_{\mathbb{K}}(Y'' \to \{\, n \,\}) \geqslant c \implies \mathcal{B}_i \cup \mathcal{S} \models (Y'' \to \{\, n \,\}).$$

Types of Implications asked

- $X \to X''$, where X is closed under $\mathcal{B}_i \cup \mathcal{S}$, but not an intent
- $X \to \{\, m \in M \mid \mathrm{conf}_{\mathbb{K}}(X \to \{\, m \,\}) \geqslant c \,\} \setminus (\mathcal{B}_i \cup \mathcal{S})(X)$, where X is an intent

Exploration by Confidence

	a	b	c
1	×	×	.
2	×	×	.
3	×	×	.
4	×	×	×
5	×	×	×
6	×	.	×
7	×	.	×
8	.	.	.
9	.	.	.
10	.	.	.

- $c := \frac{1}{2}$
- $\mathcal{S} = \{\, \{\, \mathsf{a} \,\} \to \{\, \mathsf{b} \,\} \,\}$
- p constantly $\top$

Exploration by Confidence

	a	b	c
1	×	×	.
2	×	×	.
3	×	×	.
4	×	×	×
5	×	×	×
6	×	.	×
7	×	.	×
8	.	.	.
9	.	.	.
10	.	.	.

- $c := \frac{1}{2}$
- $\mathcal{S} = \{\, \{\, \mathsf{a} \,\} \to \{\, \mathsf{b} \,\} \,\}$
- p constantly $\top$

$$\emptyset$$
$$\{\, \mathsf{a} \,\}$$
$$\{\, \mathsf{b} \,\}$$
$$\{\, \mathsf{a}, \mathsf{b} \,\}$$
$$\{\, \mathsf{c} \,\}$$
$$\{\, \mathsf{a}, \mathsf{c} \,\}$$
$$\{\, \mathsf{b}, \mathsf{c} \,\}$$

Exploration by Confidence

	a	b	c
1	×	×	.
2	×	×	.
3	×	×	.
4	×	×	×
5	×	×	×
6	×	.	×
7	×	.	×
8	.	.	.
9	.	.	.
10	.	.	.

- $c := \frac{1}{2}$
- $\mathcal{S} = \{\, \{\, \mathsf{a} \,\} \to \{\, \mathsf{b} \,\} \,\}$
- p constantly $\top$

$\emptyset$

$\{\, \mathsf{a} \,\}$

$\{\, \mathsf{b} \,\}$

$\{\, \mathsf{a}, \mathsf{b} \,\}$

$\{\, \mathsf{c} \,\}$

$\{\, \mathsf{a}, \mathsf{c} \,\}$

$\{\, \mathsf{b}, \mathsf{c} \,\}$

Exploration by Confidence

	a	b	c
1	×	×	.
2	×	×	.
3	×	×	.
4	×	×	×
5	×	×	×
6	×	.	×
7	×	.	×
8	.	.	.
9	.	.	.
10	.	.	.

- $c := \frac{1}{2}$
- $\mathcal{S} = \{ \{ \mathsf{a} \} \to \{ \mathsf{b} \} \}$
- p constantly $\top$

$$\emptyset \to \{ \mathsf{a} \}$$
$$\{ \mathsf{a} \}$$
$$\{ \mathsf{b} \}$$
$$\{ \mathsf{a}, \mathsf{b} \}$$
$$\{ \mathsf{c} \}$$
$$\{ \mathsf{a}, \mathsf{c} \}$$
$$\{ \mathsf{b}, \mathsf{c} \}$$

Exploration by Confidence

	a	b	c
1	×	×	.
2	×	×	.
3	×	×	.
4	×	×	×
5	×	×	×
6	×	.	×
7	×	.	×
8	.	.	.
9	.	.	.
10	.	.	.

- $c := \frac{1}{2}$
- $\mathcal{S} = \{ \{ \mathrm{a} \} \to \{ \mathrm{b} \} \}$
- p constantly $\top$

$\emptyset \to \{ \mathrm{a} \}$ ✓

$\{ \mathrm{a} \}$

$\{ \mathrm{b} \}$

$\{ \mathrm{a}, \mathrm{b} \}$

$\{ \mathrm{c} \}$

$\{ \mathrm{a}, \mathrm{c} \}$

$\{ \mathrm{b}, \mathrm{c} \}$

Exploration by Confidence

	a	b	c
1	×	×	.
2	×	×	.
3	×	×	.
4	×	×	×
5	×	×	×
6	×	.	×
7	×	.	×
8	.	.	.
9	.	.	.
10	.	.	.

- $c := \frac{1}{2}$
- $\mathcal{S} = \{\, \{\, a \,\} \to \{\, b \,\} \,\}$
- p constantly $\top$

$\emptyset \to \{\, a \,\}$ ✓

$\{\, a \,\}$

~~$\{\, b \,\}$~~

$\{\, a, b \,\}$

~~$\{\, c \,\}$~~

$\{\, a, c \,\}$

~~$\{\, b, c \,\}$~~

Exploration by Confidence

	a	b	c
1	×	×	.
2	×	×	.
3	×	×	.
4	×	×	×
5	×	×	×
6	×	.	×
7	×	.	×
8	.	.	.
9	.	.	.
10	.	.	.

- $c := \frac{1}{2}$
- $\mathcal{S} = \{ \{ a \} \to \{ b \} \}$
- p constantly $\top$

$\emptyset \to \{ a \}$ ✓

$\{ a \} \to \{ a, b, c \}$

~~$\{ b \}$~~

$\{ a, b \}$

~~$\{ c \}$~~

$\{ a, c \}$

~~$\{ b, c \}$~~

Exploration by Confidence

	a	b	c
1	×	×	.
2	×	×	.
3	×	×	.
4	×	×	×
5	×	×	×
6	×	.	×
7	×	.	×
8	.	.	.
9	.	.	.
10	.	.	.

- $c := \frac{1}{2}$
- $\mathcal{S} = \{ \{ \mathrm{a} \} \to \{ \mathrm{b} \} \}$
- p constantly $\top$

$\varnothing \to \{ \mathrm{a} \}$ ✓

$\{ \mathrm{a} \} \to \{ \mathrm{a}, \mathrm{b}, \mathrm{c} \}$ ✓

~~$\{ \mathrm{b} \}$~~

$\{ \mathrm{a}, \mathrm{b} \}$

~~$\{ \mathrm{c} \}$~~

$\{ \mathrm{a}, \mathrm{c} \}$

~~$\{ \mathrm{b}, \mathrm{c} \}$~~

Exploration by Confidence

	a	b	c
1	×	×	.
2	×	×	.
3	×	×	.
4	×	×	×
5	×	×	×
6	×	.	×
7	×	.	×
8	.	.	.
9	.	.	.
10	.	.	.

- $c := \frac{1}{2}$
- $\mathcal{S} = \{\{a\} \to \{b\}\}$
- p constantly $\top$

$\emptyset \to \{a\}$ ✓

$\{a\} \to \{a, b, c\}$ ✓

~~$\{b\}$~~

~~$\{a, b\}$~~

~~$\{c\}$~~

~~$\{a, c\}$~~

~~$\{b, c\}$~~

Summary

Summary

- Goal: adapt attribute exploration to handle exceptional counterexamples

Summary

- Goal: adapt attribute exploration to handle exceptional counterexamples
- Generalized attribute exploration

Summary

- Goal: adapt attribute exploration to handle exceptional counterexamples
- Generalized attribute exploration
- Developed “Exploration by Confidence”

Summary

- Goal: adapt attribute exploration to handle exceptional counterexamples
- Generalized attribute exploration
- Developed “Exploration by Confidence”

Outlook to Description Logics

Summary

- Goal: adapt attribute exploration to handle exceptional counterexamples
- Generalized attribute exploration
- Developed "Exploration by Confidence"

Outlook to Description Logics

- Implications correspond to *General Concept Inclusions* (GCIs)

Summary

- Goal: adapt attribute exploration to handle exceptional counterexamples
- Generalized attribute exploration
- Developed “Exploration by Confidence”

Outlook to Description Logics

- Implications correspond to *General Concept Inclusions* (GCIs)

$$\text{Venomous} \sqcap \text{Mammal} \sqsubseteq \bot.$$

Summary

- Goal: adapt attribute exploration to handle exceptional counterexamples
- Generalized attribute exploration
- Developed “Exploration by Confidence”

Outlook to Description Logics

- Implications correspond to *General Concept Inclusions* (GCIs)

$$\text{Venomous} \sqcap \text{Mammal} \sqsubseteq \bot.$$

- Same ideas can be applied

Summary

- Goal: adapt attribute exploration to handle exceptional counterexamples
- Generalized attribute exploration
- Developed "Exploration by Confidence"

Outlook to Description Logics

- Implications correspond to *General Concept Inclusions* (GCIs)

$$\text{Venomous} \sqcap \text{Mammal} \sqsubseteq \bot.$$

- Same ideas can be applied
- Counterexamples need to be *connected subinterpretations*

Summary

- Goal: adapt attribute exploration to handle exceptional counterexamples
- Generalized attribute exploration
- Developed "Exploration by Confidence"

Outlook to Description Logics

- Implications correspond to *General Concept Inclusions* (GCIs)

$$\text{Venomous} \sqcap \text{Mammal} \sqsubseteq \bot.$$

- Same ideas can be applied
- Counterexamples need to be *connected subinterpretations*
- Extra expert interaction required (due to growing set of "attributes")

Fact?

$$\{\,\text{Venomous}, \text{Bird}\,\} \rightarrow \bot$$