Foundations of Knowledge Representation Description Logic - Problems 2

Problem 1. Write down the following:

- 1. An unsatisfiable \mathcal{ALC} -knowledge base \mathcal{K} whose TBox is satisfiable and whose ABox contains only role assertions.
- 2. An unsatisfiable ALC-knowledge base whose TBox is empty.
- 3. An unsatisfiable ALC-TBox.
- 4. A satisfiable $\mathcal{ALC}\text{-}TBox\ \mathcal{T}$ such that all the atomic concepts occurring in \mathcal{T} are unsatisfiable w.r.t. \mathcal{T} . Write down a model of \mathcal{T} .
- 5. A satisfiable ALC-knowledge base such that all its models contain at least two domain individuals.

Hint: For this exercise, the top concept (\top) and the bottom concept (\bot) are of great help. In the first problem, for instance, you could choose as satisfiable TBox:

$$\mathcal{T} = \{\exists R. \top \sqsubseteq \bot\}$$

Problem 2. Which of the following statements is true?

- 1. $A \sqcap \neg A$ is satisfiable.
- 2. $A \sqcup \neg A$ is satisfiable.
- 3. $A \sqcap \exists r.B \sqcap \exists r.\neg B \text{ is satisfiable.}$
- *4.* $A \sqcap \exists r.B \sqcap \forall s. \neg B \text{ is satisfiable.}$
- 5. $A \sqcap \exists r.B \sqcap \forall r.\neg B \text{ is satisfiable.}$

- 6. $A \sqcap \neg A$ is subsumed by B.
- 7. B is subsumed by $A \sqcup \neg A$.
- 8. $A \sqcap \exists r.B \text{ is subsumed by } A \sqcap \exists r.\top.$
- 9. $A \sqcap \forall r.B \text{ is subsumed by } A \sqcap \exists r.B.$
- 10. $A \sqcap \exists r.B \text{ is subsumed by } A \sqcap \forall r.B.$

Problem 3. Extend the knowledge base you built in Problem 2 from the previous Exercise Sheet to capture the following statements. You may use additional features such as number restrictions if needed; you may also need more than one axiom for some of the statements.

- 1. Cars have between three and four wheels.
- 2. Bicycles have exactly two wheels.
- 3. A human who legally controls a car holds a driving license and is an adult (this is a difficult one!).
- 4. A vehicle is controlled by exactly one human.
- 5. A thing's parts' parts are that thing's parts.
- 6. A car with a broken part is broken.
- 7. Bob controls a car with a wheel that has a broken axle.