Exercise 2

SAT-Solving

Prof. Steffen Hölldobler, Emmanuelle-Anna Dietz Saldanha

International Master Program in Computational Logic — summer term 2017

20.04.2017

Exercise 2.1

1. Give the definitions as presented in the lecture for:
   - clause
   - Horn clause
   - unit clause
   - binary clause
2. Give 3 examples for each clause type.

Exercise 2.2

What is a CNF formula? What could a disjunctive normal form (DNF) be? Try to find a fast algorithm (in polynomial time) to transform a CNF into a DNF. Is this possible?

Exercise 2.3

1. Demonstrate the combinatorial explosion of the equivalences elimination given in the lecture.
2. Transform F_1, F_2 and F_3 into CNF with the help of the algorithm presented in the lecture.

$$F_1 = \neg p \wedge (p \rightarrow q) \wedge (\neg q \rightarrow p)$$
$$F_2 = \neg q \vee (\neg p \leftrightarrow q)$$
$$F_3 = p \leftrightarrow q$$

Exercise 2.4

1. An application of a rule of the form $\frac{D}{D_1 | D_2}$ may lead to copies of subformulas. May this lead to a combinatorial explosion? If this is the case, then construct a sequence of examples showing the explosion. If this is not the case, then prove it.
2. What is a definitional transformation? Why you should use it? Use the definitional transformation to transform F_1 and F_2 into a CNF.

$$F_1 = \neg q \vee (\neg p \rightarrow q)$$
$$F_2 = \neg p \wedge (p \rightarrow q) \wedge (\neg q \rightarrow p)$$

3. Transform your example of Exercise 2.4.1 with the help of the definitional transformation.

Exercise 2.5

1. What is the reduct of a formula? Give an example.
2. Let $F = \langle [1,2], [-2,3], [2,3,-4], [-2], [-1,2,-5,6], [-1,3] \rangle$. Compute the reduct of F for the following partial interpretations:
   (a) (1)
   (b) (1,2,3)
   (c) (4,5,6)
   (d) (1,2,3,4,5,6)
   (e) (-2,-3,5,6)

Exercise 2.6

1. Given the following 4 Clauses:

   $$C_1 = [1,2,3] \qquad C_2 = [-1,4]$$
   $$C_3 = [-1,2,3] \qquad C_4 = [-2,-4]$$

   Compute a resolvent of
   (a) C_1 and C_2 ,
   (b) C_1 and C_3 ,
   (c) C_1 and C_4 ,
   (d) C_3 and C_4 ,
   (e) C_2 and C_4 ,
   (f) C_2 and C_3 .
2. Let F be a CNF-formula and C a resolvent of two clauses C_1 and C_2 occurring in F , prove that $F \equiv F \wedge C$.