Technische Universität Dresden

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# Introduction to Formal Concept Analysis

## Exercise Sheet 7, Winter Semester 2017/18

**Exercise 1** (frequent concept intents and closure systems)

**Definition** (frequent concept intent)**.** *Let $\mathbb{K} = (G, M, I)$ be a formal context.*

*(a) The* support *of a set $B \subseteq M$ of attributes in $\mathbb{K}$ is given by*

$$\text{supp}(B) := \frac{|B'|}{|G|}.$$

*(b) For a given minimal support minsupp the set of* frequent concept intents *is given by*

$$\{B \subseteq M \mid \exists A \subseteq G : (A, B) \in \mathfrak{B}(G, M, I) \wedge \text{supp}(B) \geq \mathit{minsupp}\}.$$

Show that the set of frequent concept intents together with the set $M$ forms a closure system.

**Solution:**

**Proof:** We have to show that the intersection of frequent concept intents is again a frequent concept intent. We already know that the intersection of intents produces an intent. It remains to show that it is frequent. So let $\mathfrak{I}$ be a set of frequent intents. We pick one $B \in \mathfrak{I}$ and observe $\text{supp}(B) = \frac{|B'|}{|G|} \geq \mathit{minsupp}$. Moreover, we have $\bigcap \mathfrak{I} \subseteq B$ and consequently $B' \subseteq (\bigcap \mathfrak{I})'$. Therefore $\text{supp}(\bigcap \mathfrak{I}) = \frac{|(\bigcap \mathfrak{I})'|}{|G|} \geq \frac{|B'|}{|G|} \geq \mathit{minsupp}$, i.e., $\bigcap \mathfrak{I}$ is frequent.

**Exercise 2** (support)

Show the validity of the properties of the support function that are employed by the TITANIC algorithm:

Let $(G, M, I)$ be a formal context $X, Y \subseteq M$. Then it holds:

1) $X \subseteq Y \implies \text{supp}(X) \geq \text{supp}(Y)$

2) $X'' = Y'' \implies \text{supp}(X) = \text{supp}(Y)$

3) $X \subseteq Y \wedge \text{supp}(X) = \text{supp}(Y) \implies X'' = Y''$

**Solution:**

1. Let $X \subseteq Y$, then $Y' \subseteq X'$ holds as we saw in Exercise Sheet 1. This implies, $\text{supp}(Y) = \frac{|Y'|}{|G|} \leq \frac{|X'|}{|G|} = \text{supp}(X)$
2. $X'' = Y'' \implies \text{supp}(X) = \text{supp}(Y)$
   $X'' = Y'' \iff X''' = Y''' \iff X' = Y' \implies \text{supp}(X) = \frac{|X'|}{|G|} = \frac{|Y'|}{|G|} = \text{supp}(|Y|)$.
3. $X \subseteq Y \wedge \text{supp}(X) = \text{supp}(Y) \implies X'' = Y''$
   $\text{supp}(X) = \text{supp}(Y) \implies |X'| = |Y'|$ and $X \subseteq Y \implies X' \supseteq Y'$. Hence $X' = Y'$, since $X'$ and $Y'$ are finite. It follows, $X'' = Y''$.

**Exercise 3** (computing concept intents with TITANIC)

The following context contains transactions in a supermarket. Compute the closure system of all concept intents using the TITANIC algorithm. (hint: use the table structure from the example computation in the lecture slides)

| | apples (a) | beer (b) | chips (c) | tv magazine (d) | toothpaste (e) |
|---|---|---|---|---|---|
| $t_1$ | × | × | × | | |
| $t_2$ | | | × | × | |
| $t_3$ | | × | × | × | |
| $t_4$ | × | × | | | × |
| $t_5$ | | | × | | × |
| $t_6$ | | × | × | × | |
| $t_7$ | × | × | | | |
| $t_8$ | | | × | × | |

**Solution:**

In the first pass, the algorithm deals with the empty set and singletons, all 1-sets. It returns the results for k =0 and k=1:

k = 0:

| step 1 | | step2 |
|---|---|---|
| x | x.s | $x \in k_k$ ? |
| ∅ | 1 | yes |

k = 1:

| steps 4+5 | | step 7 | step 9 |
|---|---|---|---|
| $X$ | $X.p\_s$ | $X.s$ | $X \in K_k$? |
| {a} | 1 | 3/8 | yes |
| {b} | 1 | 5/8 | yes |
| {c} | 1 | 6/8 | yes |
| {d} | 1 | 4/8 | yes |
| {e} | 1 | 2/8 | yes |

Step 8 returns: ∅.closure ← ∅

k = 2:

| steps 12 | | step 7 | step 9 |
|---|---|---|---|
| $X$ | $X.p\_s$ | $X.s$ | $X \in K_k$? |
| {a,b} | 3/8 | 3/8 | no |
| {a,c} | 3/8 | 1/8 | yes |
| {a,d} | 3/8 | 0 | yes |
| {a,e} | 2/8 | 1/8 | yes |
| {b,c} | 5/8 | 3/8 | yes |
| {b,d} | 4/8 | 2/8 | yes |
| {b,e} | 2/8 | 1/8 | yes |
| {c,d} | 4/8 | 4/8 | no |
| {c,e} | 2/8 | 1/8 | yes |
| {d,e} | 2/8 | 0 | yes |

Step 8 returns:
$\{a\}$.closure $\leftarrow \{a, b\}$
$\{b\}$.closure $\leftarrow \{b\}$
$\{c\}$.closure $\leftarrow \{c\}$
$\{d\}$.closure $\leftarrow \{c, d\}$
$\{e\}$.closure $\leftarrow \{e\}$

k = 3:

| steps 12 | | step 7 | step 9 |
|---|---|---|---|
| $X$ | $X.p\_s$ | $X.s$ | $X \in K_k$? |
| {a,c,e} | 1/8 | 0 | yes |
| {a,d,e} | 0 | 0 | no |
| {b,c,e} | 1/8 | 0 | yes |
| {b,d,e} | 0 | 0 | no |

Step 8 returns:
$\{a, c\}$.closure $\leftarrow \{a, b, c\}$
$\{a, d\}$.closure $\leftarrow \{a, b, c, d, e\}$
$\{a, e\}$.closure $\leftarrow \{a, b, e\}$
$\{b, c\}$.closure $\leftarrow \{b, c\}$
$\{b, d\}$.closure $\leftarrow \{b, c, d\}$
$\{b, e\}$.closure $\leftarrow \{a, b, e\}$
$\{c, e\}$.closure $\leftarrow \{c, e\}$
$\{d, e\}$.closure $\leftarrow \{a, b, c, d, e\}$

k= 4:

Step 12: returns the empty set. Hence there is nothing to **WEIGH** in Step 7. Step 9 sets k_4 = $\emptyset$; and in step 10, the loop is exited.
Step 8 returns:
$\{a, c, e\}$.closure $\leftarrow \{a, b, c, d, e\}$
$\{b, c, e\}$.closure $\leftarrow \{a, b, c, d, e\}$

Step 14: Collects all concept intents: