# Foundations of Knowledge Representation
# Uncertainty Problems

**Problem 1.** *From the probability axioms, we can deduce several further rules for computing probabilities. Consider the following two:*

$R_1$ $P(\neg A) = 1 - P(A)$.

$R_2$ $P(A) = 0$, *if* $A$ *is a contradiction.*

*Derive these from the Kolmogorov axioms.*

**Problem 2.** *Suppose that* $W = \{w_1, w_2, w_3\}$. *Define* $m$ *as follows:*

$$\begin{aligned} m(w_1) &= 1/4 \\ m(w_1, w_2) &= 1/4 \\ m(w_2, w_3) &= 1/2 \\ m(U) &= 0 \text{ if } U \text{ is not one } \{w_1\}, \{w_1, w_2\}, \text{ or } \{w_2, w_3\}. \end{aligned}$$

Provide all values for:

$Bel_m(w_1)$, $Bel_m(w_2)$, $Bel_m(w_3)$, $Bel_m(w_1, w_2)$, $Bel_m(w_1, w_3)$, $Bel_m(w_2, w_3)$, $Bel_m(w_1, w_2, w_3)$

as well as

$Plaus_m(w_1)$,$Plaus_m(w_2)$,$Plaus_m(w_3)$,$Plaus_m(w_1, w_2)$, $Plaus_m(w_1, w_3)$, $Plaus_m(w_2, w_3)Plaus_m(w_1, w_2, w_3)$

**Problem 3.** *From the lecture (slide 33), recall the following result for combining the masses of two tests:*

$$\begin{aligned} &(m_1 \oplus m_2)(\{hep\}) = 0.8 \\ &(m_1 \oplus m_2)(\{hep, cirr\}) = 0.12 \\ &(m_1 \oplus m_2)(W) = 0.08 \end{aligned}$$

*Illustrate each step of the computation to derive the above result.*

**Problem 4.** *In a multi-sensor system, we need to combine evidences from multiple sensors. Assume that we have two sensors and three observable events with* $W = A, B, C$. *From the sensors, we get the following pieces of information that indicate the likelihood of observing each event:*

$Sensor_1$ $m_1(A) = 0.95, m_1(B) = 0.05, m_1(C) = 0$

$Sensor_2$ $m_2(A) = 0, m_2(B) = 0.1, m_2(C) = 0.9$

*Do the following:*
*(i) Compute* $m_1 \oplus m_2(A)$, $m_1 \oplus m_2(B)$, $m_1 \oplus m_2(C)$,
*(ii) Argue whether or not your result poses a problem for the combination rule.*

**Problem 5.** *The sorites paradox can be seen as one of the most striking paradoxes for logical reasoning (Hájek and Novák: 2003). Consider this paradox in the following form:*

*(1)* $10^{100}$ *is a huge number.*

*(2) If* $n$ *is a huge number, then* $n - 1$ *is also huge.*

*From this, deduce:*

*(3) 0 is a huge number.*

*Do the following: (i) Describe the paradox. (ii) Argue how fuzzy logic could be helpful to resolve the paradox given the following intuition: It seems that* (2) *is not fully, but only almost true.*