Bounded Treewidth and the Infinite Core Chase

Complications and Workarounds toward Decidable Querying

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Query Answering with Existential Rules

Basic Problem (Entailment)

INPUT: *a knowledge base* $K = (F, \Sigma)$ *where* F *is a database instance and* Σ *is a finite set of existential rules, a (Boolean) conjunctive query* Q.

QUESTION: *does* $\mathcal{K}$ *entail* Q*, i.e.* $F, \Sigma \models Q$*?*

1. **Atomset:** a *possibly infinite (countable)* set of atoms over constants and variables (no equality, no function symbol);
2. **Database (Instance):** a *finite* atomset;
3. **(Existential) Rule (or tgd):** a pair of *finite* atomsets whose general logic form is $\forall\vec{X}\,\forall\vec{Y}\,(body(\vec{X},\vec{Y}) \rightarrow \exists\vec{Z}\,head(\vec{Y},\vec{Z})))$;
4. **(Boolean Conjunctive) Query**: a *finite* atomset.

Object	Example	Logical form
Database	$p(a,b), q(b,c)$	$p(a,b) \wedge q(b,c)$
Rule	$p(X,Y), q(Y,Z) \rightarrow r(X,T,Z), s(T)$	$\forall X \forall Y \forall Z\,(p(X,Y) \wedge q(Y,Z) \rightarrow \exists T\,(r(X,T,Z) \wedge s(T)))$
Query	$r(a,U,b), p(a,V), q(W,b)$	$\exists U \exists V \exists W\,(r(a,U,b) \wedge p(a,V) \wedge q(W,b))$

The Great AIJ Blunder of 2011 (**BagLecMugSal11**)

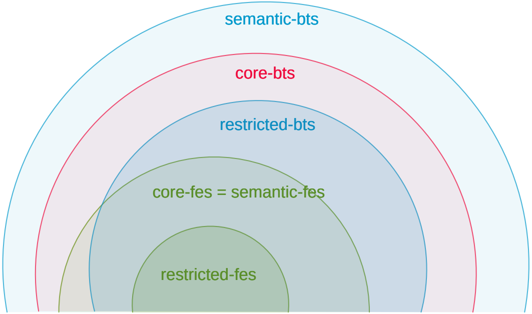


Figure: A cartography of some (abstract) decidable subclasses for the entailment problem, following (**BagLecMugSal11**)

1. **Preliminaries:** why is the proof of decidability for *core-bts* wrong ?
2. **The steepening staircase:** the class *core-bts* is misplaced !
3. **Robust aggregations:** a new proof of decidability for the *core-bts* class.
   - a novel way to define the result of an infinite chase;
   - the need to consider *finitely-universal* models instead of the usual *universal models*.

Preliminary Notions

Semantic Definition of Finite Expansion Sets (fes)

1. **Trigger:** a (K-)*trigger* for an atomset F is a pair $t = (R, \pi)$ where $R \in \Sigma$ and π maps *body*(R) into F. It is *satisfied* when π extends to map *body*$(R) \cup$ *head*(R) into F.
2. **Model:** an atomset I (seen as an interpretation) is a *model* (of K) when it is a model of F and all K-triggers for I are satisfied.
3. **Universality:** an atomset is *universal* (for K) when it maps to every model of K.
4. **(BCQ) Representative:** a (BCQ)-*representative* of K is an atomset I such that, for any Q, we have $K \models Q \Leftrightarrow I \models Q$.

Theorem (Universal Models)

If an atomset is a universal model of K, then it is a BCQ representative of K.

5. **Semantic fes:** a set of rules Σ belongs to the (decidable but unrecognizable) *semantic fes* class when, for every F, (F, Σ) admits a finite universal model.

Semantic Definition of Bounded Treewidth Sets (bts)

Theorem (Treewidth and decidability, (Cou90) + (BagLecMugSal11))

Entailment is decidable for KBs admitting a universal model of finite treewidth.

2. **Treewidth:** the *treewidth* of an atomset F measures its similarity to a tree. If F is a tree, then $tw(F) = 1$. If F contains a grid of unbounded size, then $tw(F) = +\infty$.
3. **Semantic bts:** a set of rules Σ belongs to the (decidable) *semantic bts* class when, for every F, (F, Σ) admits an universal model of finite treewidth.
4. semantic-fes $\subset$ semantic-bts

Theorem (Compactness of treewidth, Thomas88thetree-width)

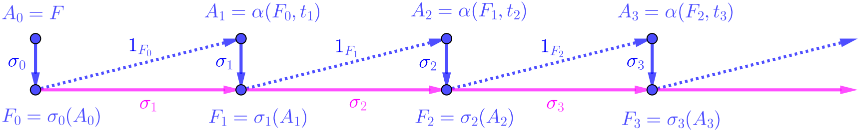
If every finite subset B of an atomset A has treewidth $tw(B) \leq k$, then $tw(A) \leq k$.

Derivations and their Results (1)

1. **Rule application:** let $t = (R, \pi)$ be a trigger in F. Then the *application* of t on F produces the atomset $\alpha(F, t) = F \cup \pi^{safe}(head(R))$.

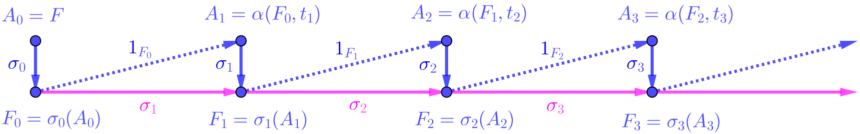
Proposition (Properties of Rule Application)

A trigger t for F is satisfied in $\alpha(F, t)$. Moreover, if F is universal, then $\alpha(F, t)$ is universal.



2. **Derivation:** a *possibly infinite* sequence $\mathcal{D} = (F_i)$ where $F_0 = \sigma_0(F)$ and $F_i = \sigma_i(\alpha(F_{i-1}, t_i))$, the σ_i being endomorphisms.
3. **Fairness:** $\mathcal{D}$ is said *fair* when, for any trigger (R, π) for some F_i, there is some F_j in which $(R, \sigma_i^j \circ \pi)$ is satisfied.

Derivations and their Results (2)



1. **Finite result:** if $\mathcal{D}$ is a finite derivation, then its *finite result* $\mathcal{D}^+$ is its last atomset.
2. **Natural aggregation:** the *natural aggregation* of a (possibly infinite) derivation $\mathcal{D} = (F_i)_{i \in \mathfrak{I}}$ is the (possibly infinite) atomset $\mathcal{D}^* = \cup_{i \in \mathfrak{I}} F_i$.

Theorem (Finite result)

The finite result $\mathcal{D}^+$ of a fair derivation is a finite universal model.

Theorem (Natural aggregation)

The natural aggregation $\mathcal{D}^*$ of a fair monotonic derivation $\mathcal{D}$ is a (possibly infinite) universal model. If $\mathcal{D}$ is non-monotonic, then $\mathcal{D}^*$ is a universal BCQ representative, but not necessarily a model.

The Restricted and Core Chases: the Terminating Case

1. **Restricted chase (fkmp05):** the *restricted chase* is a fair derivation that only applies unsatisfied triggers and whose endomorphisms σ_i are the identity. The restricted chase is *monotonic*.
2. **Core:** a *finite* atomset is a *core* when there is no homomorphism into one of its strict subsets. Every finite atomset maps to a subset which is a core.
3. **Core chase (DBLP:conf/pods/DeutschNR08):** the *core chase* is a fair derivation that only applies unsatisfied triggers and whose endomorphisms σ_i map A_i to a core. The core chase is not always monotonic.
4. **Restricted-fes:** a set of rules Σ belongs to the (decidable) *restricted-fes* class when, for every F, the restricted chase halts on (F, Σ).
5. **Core-fes:** a set of rules Σ belongs to the (decidable) *core-fes* class when, for every F, the core chase halts on (F, Σ).
6. restricted-fes $\subset$ core-fes = semantic-fes

The Restricted and Core Chases: the Bounded Treewidth Case

1. **Bounded treewidth:** a derivation $\mathcal{D}$ has *bounded treewidth* k when $\forall F_i$, $tw(F_i) \leq k$.

Theorem (Treewidth and monotonic derivations)

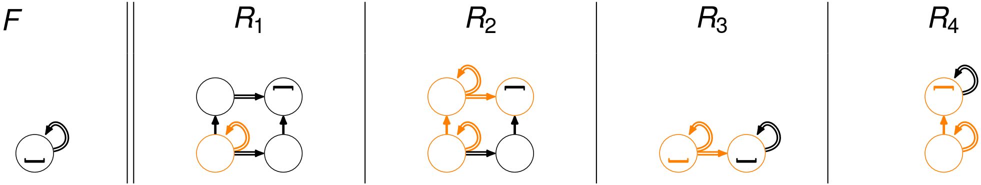
If $\mathcal{D}$ is a monotonic derivation with bounded treewidth k, then $tw(\mathcal{D}^*) \leq k$.

1. the *natural aggregation* $\mathcal{D}^*$ of a **restricted chase** is a universal model.
2. the *natural aggregation* $\mathcal{D}^*$ of a **restricted chase** of bounded treewidth is an atomset of finite treewidth.
3. **Restricted-bts:** a set of rules Σ belongs to the (decidable) *restricted-bts* class when, for every F, the restricted chase from (F, Σ) has bounded treewidth.

1. the *natural aggregation* $\mathcal{D}^*$ of a **core chase** is not necessarily a model.
2. the *natural aggregation* $\mathcal{D}^*$ of a **core chase** of bounded treewidth may not have finite treewidth.
3. **No reason for core-bts decidability:** a set of rules Σ belongs to the *core-bts* class when, for every F, the core chase from (F, Σ) has bounded treewidth.

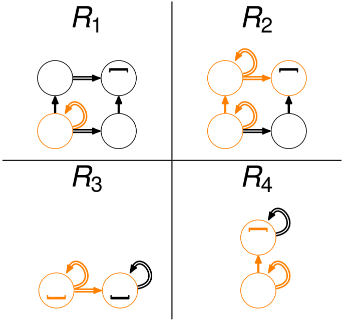
The Steepening Staircase

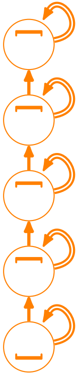
The Steepening Staircase: Presentation of the KB



1. **Facts:** $d(X), h(X, X)$.
2. **Rules:**
   1. $h(X, X) \rightarrow \exists Y \exists Z \exists T\, (v(X, Y), h(Y, Z), u(Z), v(T, Z), h(X, T))$.
   2. $h(X, X), v(X, Y), h(Y, Y), h(Y, Z) \rightarrow \exists T\, h(X, T), v(T, Z), u(Z)$.
   3. $d(X), h(X, X), h(X, Y) \rightarrow d(Y), h(Y, Y)$.
   4. $h(X, X), v(X, Y), u(Y) \rightarrow h(Y, Y)$.

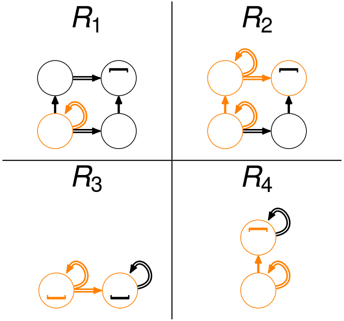
The Steepening Staircase: Elementary Step of a Derivation

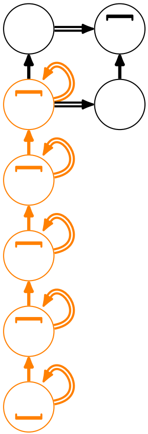




1. **Elementary step of the derivation:** from a C_k we build a S_k, containing a C_{k+1}

The Steepening Staircase: Elementary Step of a Derivation





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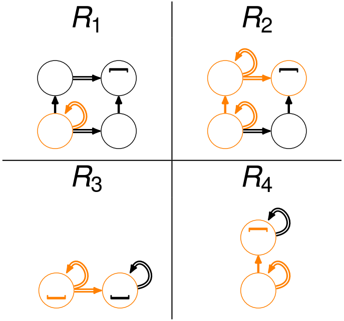

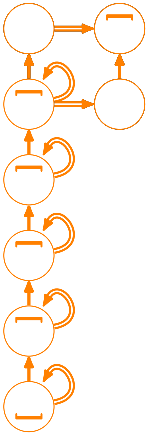

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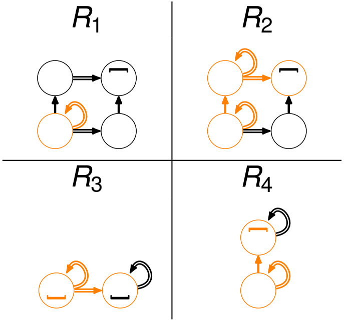

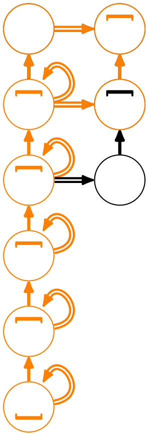

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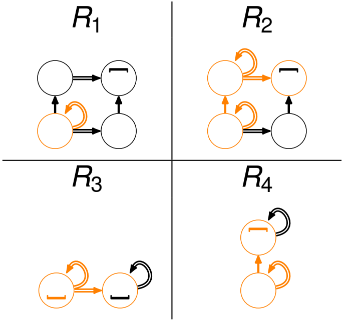

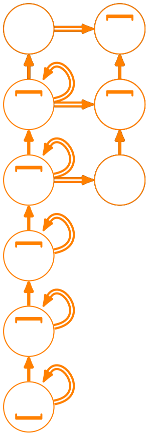

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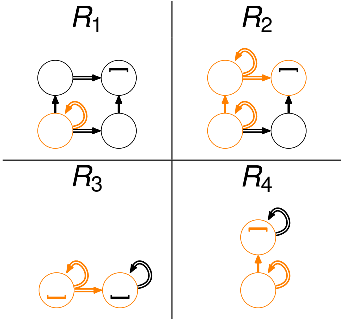

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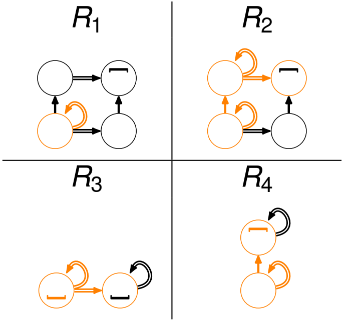

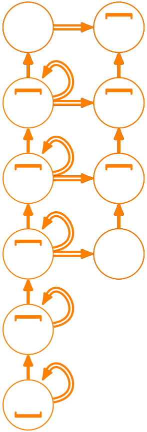

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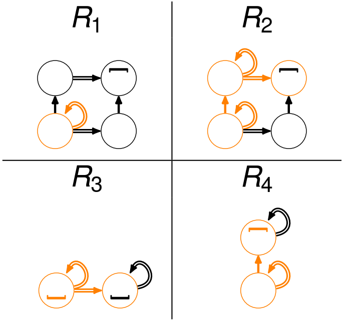

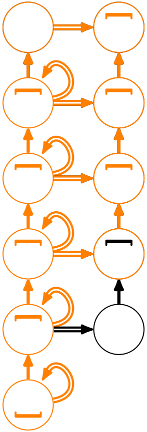

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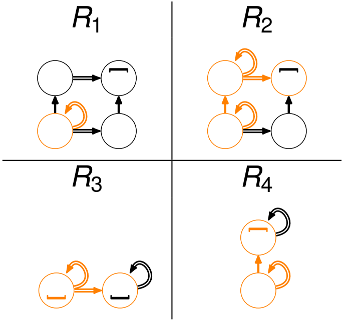

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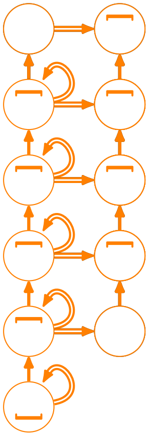

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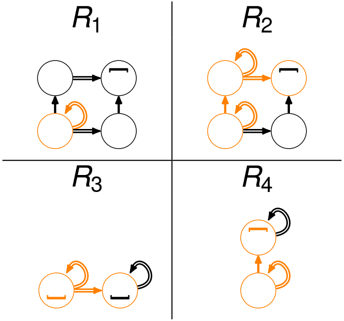

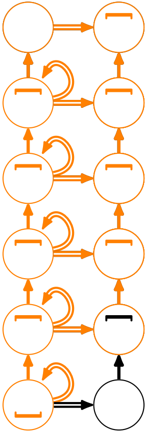

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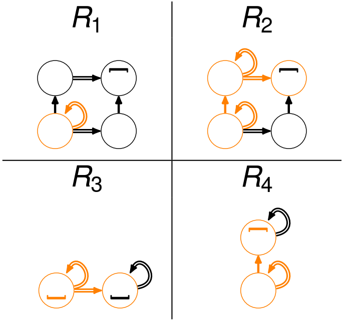

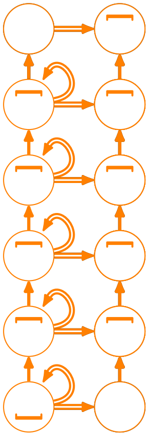

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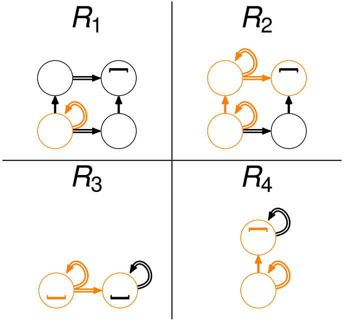

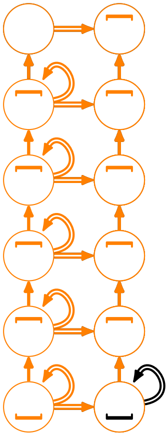

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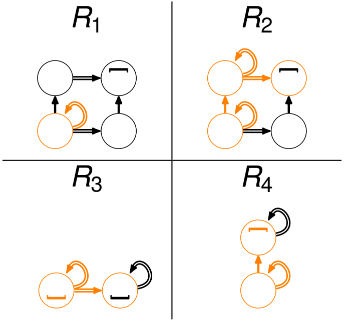

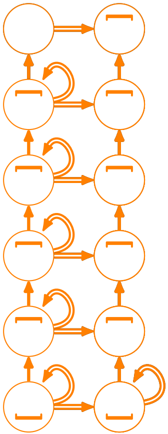

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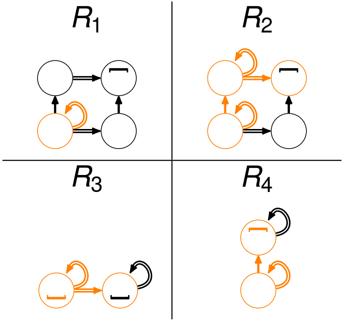

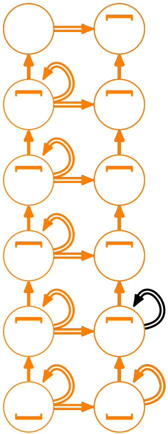

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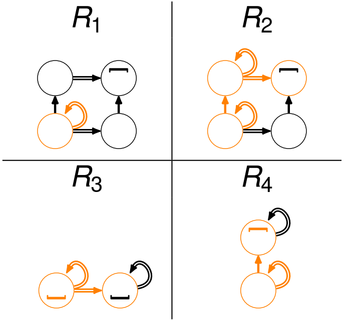

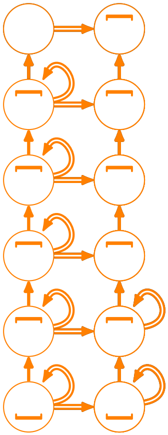

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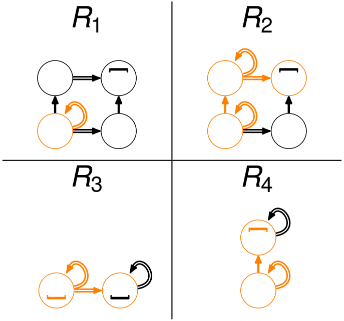

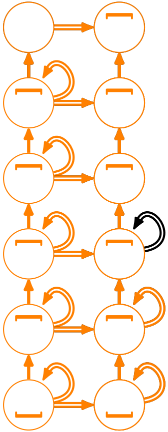

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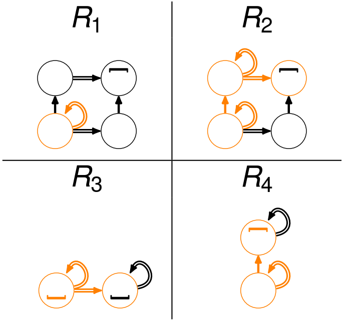

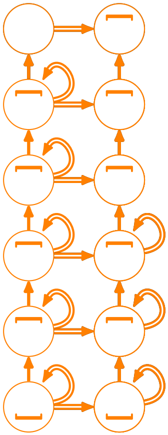

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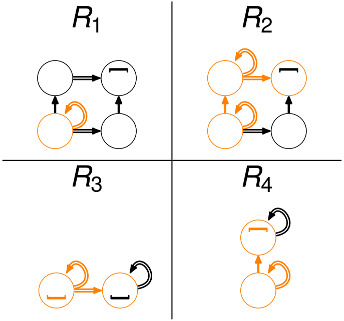

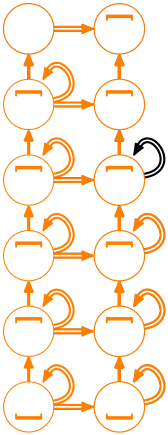

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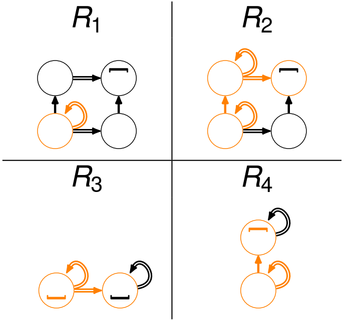

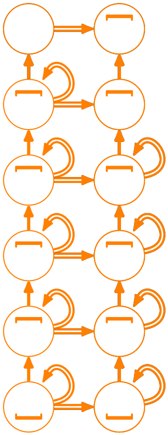

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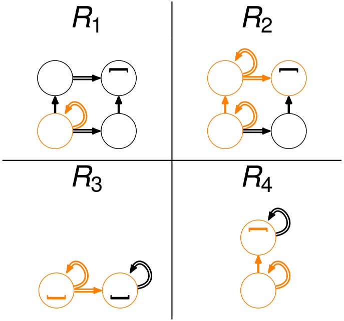

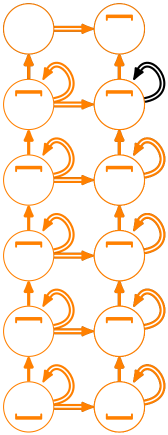

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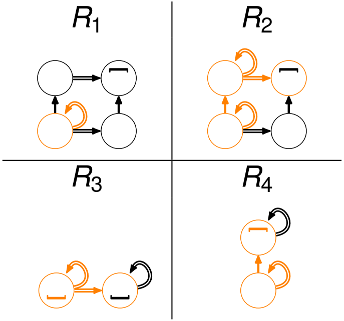
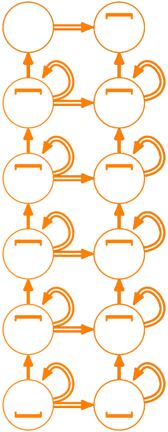
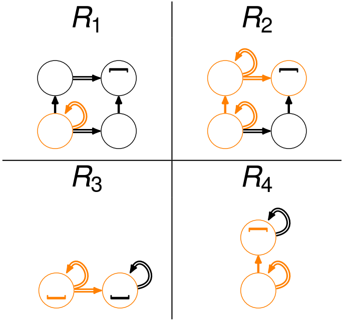
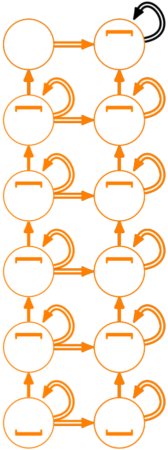




1. **Elementary step of the derivation:** from a C_k we build a S_k, containing a C_{k+1}

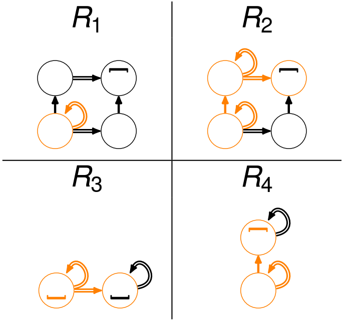
The Steepening Staircase: Elementary Step of a Derivation

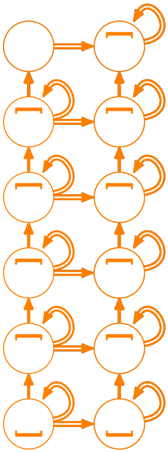




1. **Elementary step of the derivation:** from a C_k we build a S_k, containing a C_{k+1}

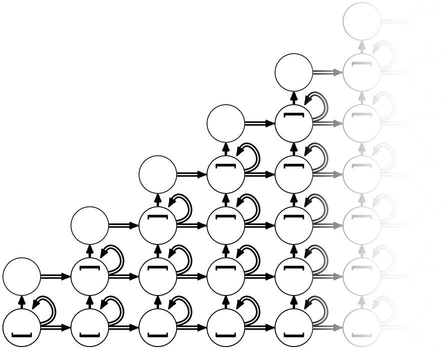
The Steepening Staircase: Elementary Step of a Derivation





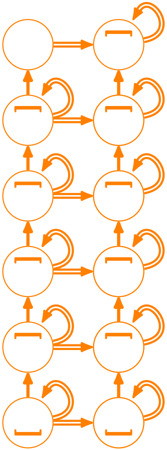
1. **Elementary step of the derivation:** from a C_k we build a S_k, containing a C_{k+1}

The Steepening Staircase: Universal Models of Infinite Treewidth



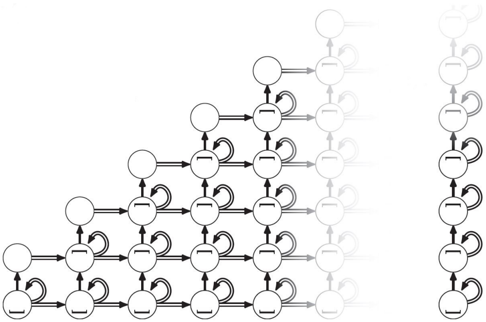
1. **Restricted chase:** the natural aggregation of a restricted chase $\mathcal{D}^*$ is a universal model of infinite treewidth (grids of unbounded size).
2. **Moreover,** every universal model of the steepening staircase KB has infinite treewidth (see paper).

The Steepening Staircase: a Core Chase of Bounded Treewidth



1. **Core of a $\mathbf{S_k}$:** the core of a S_k is a C_{k+1}. All atomsets built from a C_k to S_k are cores.
2. **Core chase:** the atomsets along the core chase have treewidth between 1 and 2.
3. **Treewidth:** the natural aggregation of the core chase is the same as the one obtained from the restricted chase, and has thus infinite treewidth.
4. **Consequence:** core-bts $\not\subset$ semantic-bts

Finitely-Universal Models



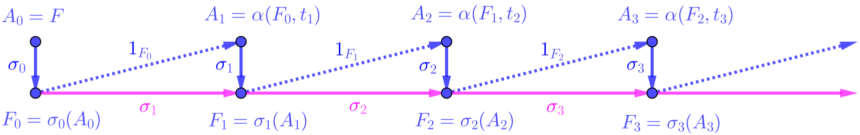
1. the infinite column C_∞ is a model of the steepening staircase KB, but it is not universal.
2. **Finite-universality:** an atomset A is *finitely-universal* when every subset of A is universal.
3. C_∞ is finitely-universal.

Theorem ((BCQ) representative)

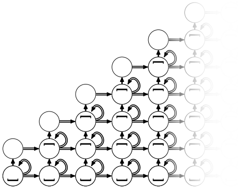
A finitely-universal model of a KB $\mathcal{K}$ is a (BCQ) representative of $\mathcal{K}$.

Robust Aggregations

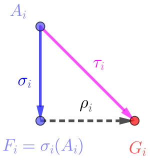
Robust Renaming



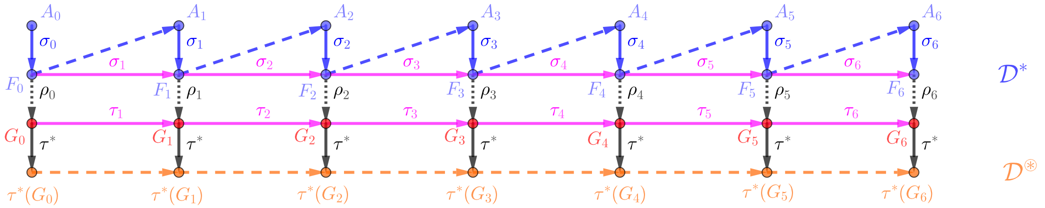
1. We want to define the **result** as $\cup_{i \in \mathfrak{I}} \sigma^*(F_i)$, but it doesn't work !



1. **Robust renaming:** if $X \in vars(F_i)$, we define $\rho_i(X) = min(\sigma_i^{-1}(X))$
2. ρ_i is an **isomorphism** and $\tau_i = \rho_i \circ \sigma_i$ is such that, for any $X \in vars(A_i)$, $\tau_i(X) \leq X$.



Robust Aggregation



1. Apply the robust renaming all along the derivation. See that $\tau_i = \rho_i \circ \sigma_i$ is also a homomorphism from G_{i-1} to G_i, and that G_i is isomorphic to F_i.
2. **The τ_i are finitely morphing:** if X is a variable in F_i, there is $j \geq i$ such that for any $r \geq j$, $\tau_i^j(X) = \tau_i^r(X)$. We can thus **define** $\tau^*(X) = \tau_i^j(X)$.
3. **Robust aggregation:** if $\mathcal{D}$ is a derivation, we call $\mathcal{D}^\circledast = \cup_{i \in \mathfrak{I}} \tau^*(G_i)$ its *robust aggregation*.

Main Properties of Robust Aggregation

Theorem (Model)

$\mathcal{D}^{\circledast}$ is a model.

1. $\mathcal{D}^{*}$ is not a model for nonmonotonic derivations.

Theorem (Finite-universality)

$\mathcal{D}^{\circledast}$ is finitely-universal.

2. $\mathcal{D}^{*}$ is always universal.

Theorem

If $\mathcal{D}$ is a derivation with bounded treewidth k, then $\mathcal{D}^{\circledast}$ has treewidth $\leq k$.

3. $\mathcal{D}^{*}$ may have infinite treewidth (see steepening staircase).

Finishing touches

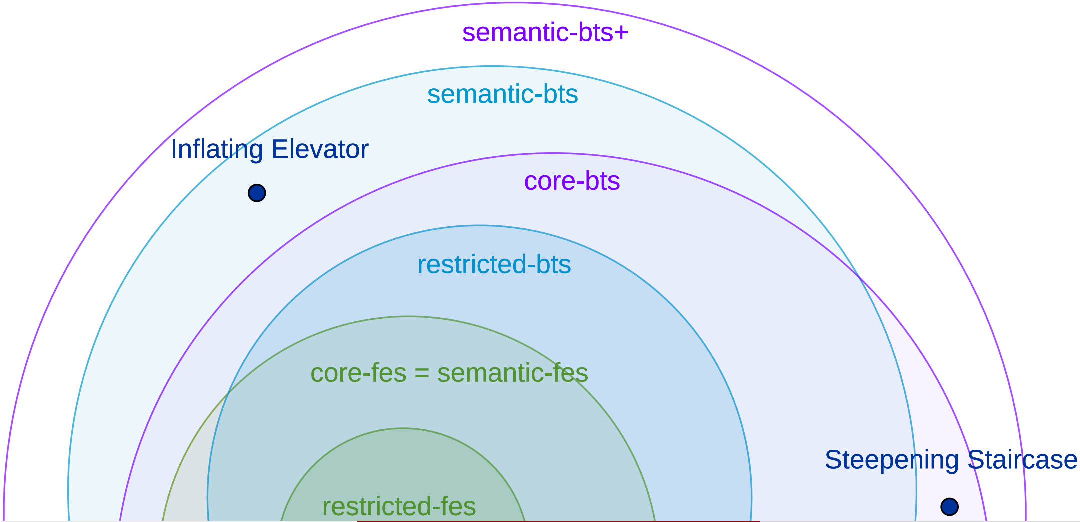
1. If $\mathcal{D}$ is a chase with bounded treewidth k, then $\mathcal{D}^{\circledast}$ is a finitely-universal model (and thus a BCQ representative) of treewidth $\leq k$.

Theorem (Basically (**Cou90**) + (**BagLecMugSal11**) + (this paper))

CQ entailment is decidable for KBs admitting a finitely-universal model of finite treewidth.

2. **Semantic-bts+:** a set of rules Σ belongs to the (decidable) *semantic-bts+* class when, for every F, (F, Σ) admits a finitely-universal model of finite treewidth.
3. this is now a proof of decidablity of **core-bts** !

The New Map of Abstract Decidable Classes

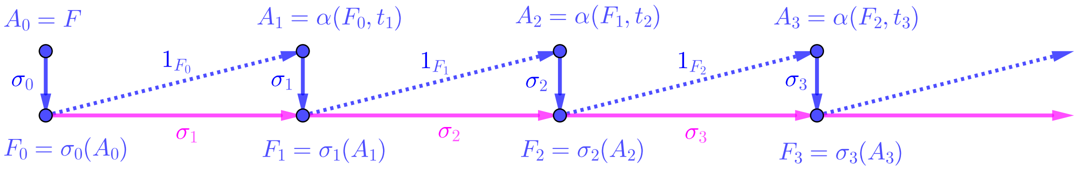


References I

Part I

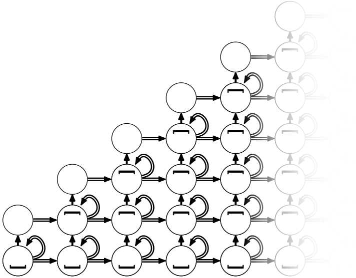
Appendix

Nonmonotonic derivations: nothing is lost [. . .] everything is transformed

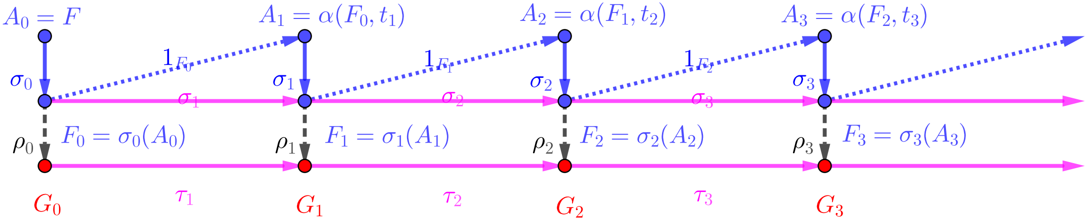


1. **The Lavoisier point of view:** If a is an atom in some F_i, it may not be in F_j (with $j > i$), but it has not disappeared: it has morphed to $\sigma_i^j(a)$ in F_j.

2. **Problem:** what is the result of the morphing of a in $\mathcal{D}^*$?

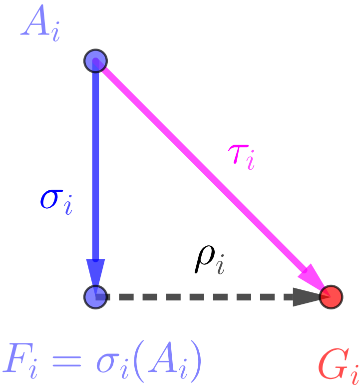


Our strategy: ensuring variables to be finitely morphing



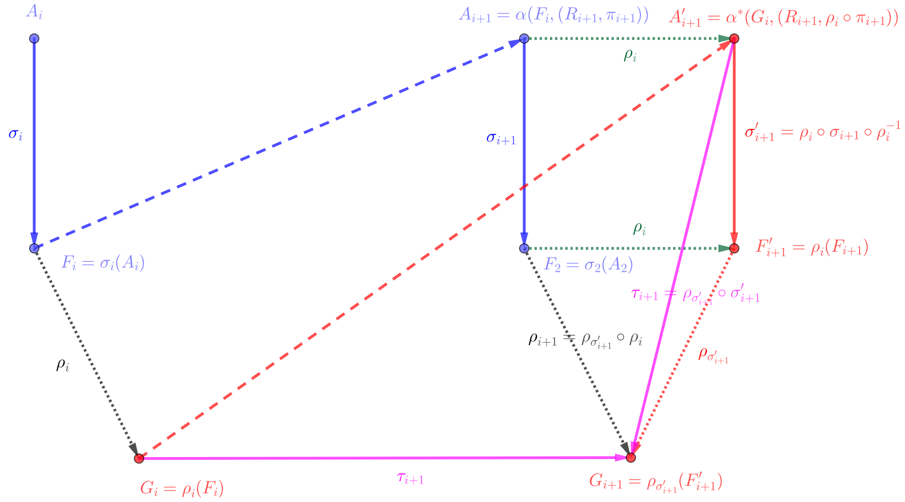
1. **Renaming variables of F_i:** each G_i is isomorphic to F_i (with isomorphism ρ_i), and we build homomorphisms τ_i from G_{i-1} to G_i.
2. **The τ_i are finitely morphing:** if X is a variable in F_i, there is $j \geq i$ such that for any $r \geq j$, $\tau_i^j(X) = \tau_i^r(X)$. We can thus **define** $\tau^*(X) = \tau_i^j(X)$.
3. **Robust aggregation:** if $\mathcal{D}$ is a derivation, we call $\mathcal{D}^{\circledast} = \cup_{i \in \mathfrak{I}} \tau^*(G_i)$ its *robust aggregation*.

How to reach that goal (1): the renaming operation

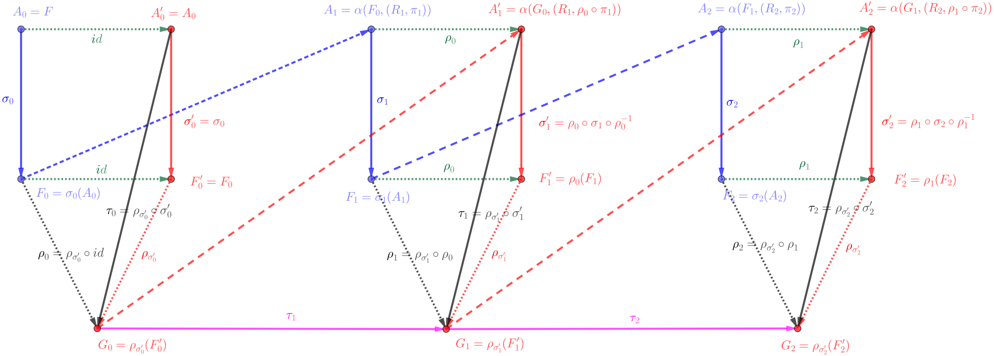


1. **Total ordering of variables:** bijection $s : \mathcal{X} \to \mathbb{N}$. We note $X \leq_{\mathcal{X}} Y$ when $s(X) \leq s(Y)$. As a consequence, $(\mathcal{X}, \leq_{\mathcal{X}})$ is **well-founded.**
2. **The renaming:** if $F_i = \sigma_i(F_i)$ (where σ_i is a retraction), and X a variable of F_i, we define $\rho_i(X) = min_{\mathcal{X}}(\sigma_i^{-1}(X))$
3. **Important properties:** ρ_i is an **isomorphism** and the homomorphism $\tau_i = \rho_i \circ \sigma_i$ is such that, for any variable X in A_i, $\tau_i(X) \leq_{\mathcal{X}} X$.

How to reach that goal (2): building the G_i



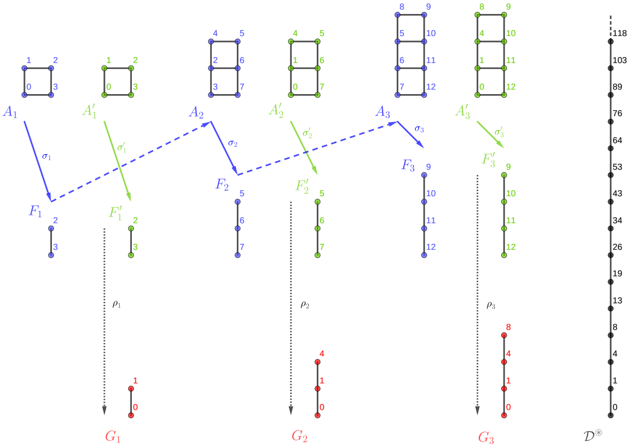
How to reach that goal (3): wrapping it up



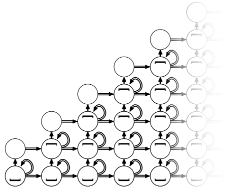
Theorem (Finitely morphing, monotonicity)

The τ_i are finitely morphing, allowing to define τ^*. The $\tau^*(G_i)$ are **monotonic**.

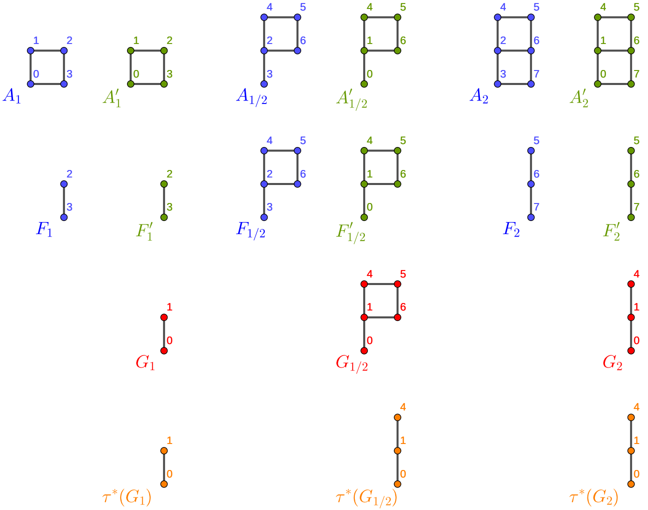
How about a staircase example?



1. **Model:** $\mathcal{D}^{\circledast}$ is a *model* of the staircase KB.
2. **Universality:** $\mathcal{D}^{\circledast}$ is not universal. However, its is **finitely universal.**
3. **Treewidth:** $\mathcal{D}^{\circledast}$ has treewidth 1.



The staircase example: a closer look



Main properties of robust aggregation

Theorem (Model)

$\mathcal{D}^{\circledast}$ is a model.

1. $\mathcal{D}^{*}$ is not a model for nonmonotonic derivations.

Theorem (Finite universality)

$\mathcal{D}^{\circledast}$ is finitely universal.

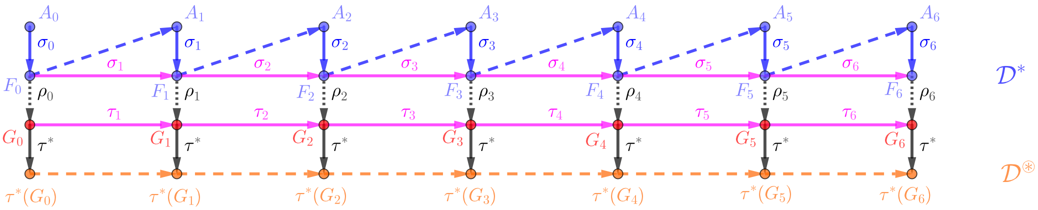
2. $\mathcal{D}^{*}$ is always universal.

Theorem ((BCQ) representative)

A finitely universal model of a KB $\mathcal{K}$ is a (BCQ) representative of $\mathcal{K}$.

3. The natural aggregation $\mathcal{D}^{*}$ is also a (BCQ) representative.

Preliminary: a kind of monotonicity

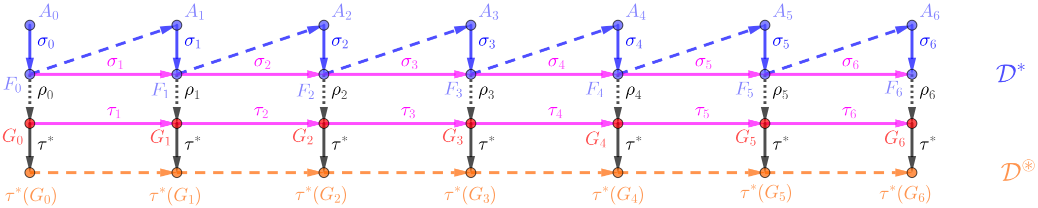


1. **Where is Wally ?** if $(B_i)_{i\in\mathfrak{I}}$ is a **monotonic** sequence of atomsets, then for any **finite** subset A of $\cup_{i\in\mathcal{I}}B_i$, there exists $j \in \mathfrak{I}$ such that, for any $k \geq j \in \mathfrak{I}$, $A \subseteq B_k$.
2. **Monotonic derivations:** if $\mathcal{D} = (F_i)_{i\in\mathfrak{I}}$ is a monotonic derivation and A is a finite subset of $\mathcal{D}^*$, then there exists $j \in \mathfrak{I}$ such that, for any $k \geq j \in \mathfrak{I}$, $A \subseteq F_k$.

Lemma (Where is Wally)

If A is a finite subset of $\mathcal{D}^{\circledast}$, then there exists $j \in \mathfrak{I}$ such that, for any $k \geq j \in \mathfrak{I}$, $A \subseteq G_k$.

$\mathcal{D}^{\circledast}$ is finitely universal

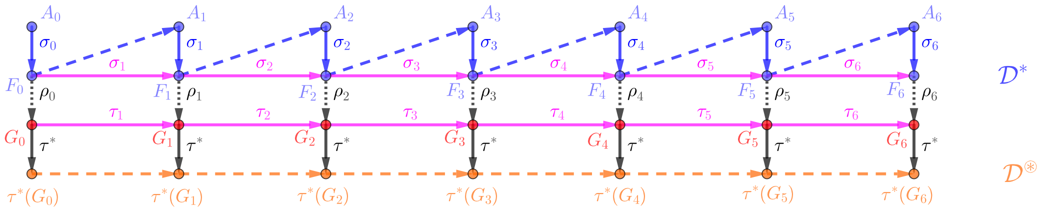


Theorem (Finite universality)

$\mathcal{D}^{\circledast}$ *is finitely universal.*

1. if A is a finite subset of $\mathcal{D}^{\circledast}$, then there exists G_i such that $A \subseteq G_i$ (Wally lemma).
2. since G_i is isomorphic to F_i and F_i universal, then G_i is universal.
3. as a subset of a universal atomset, A is universal □

$\mathcal{D}^{\circledast}$ is a model (1)

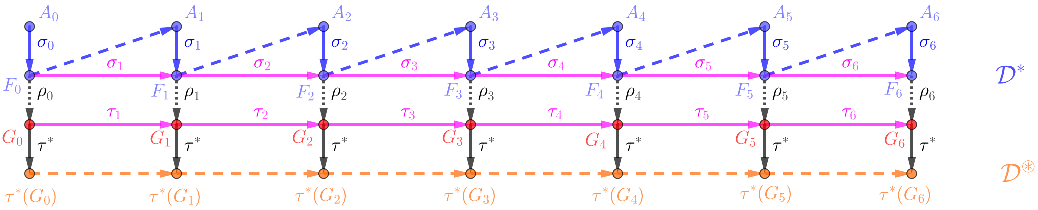


Theorem (Model)

$\mathcal{D}^{\circledast}$ is a model.

1. if (R, π) is a trigger for $\mathcal{D}^{\circledast}$, then $\pi(body(R))$ is a finite subset of $\mathcal{D}^*$ and thus (Wally lemma) there is G_i such that (R, π) is a trigger for G_i.
2. then $(R, \rho_i^{-1} \circ \pi)$ is a trigger for F_i, and by **fairness** $(R, \sigma_i^j \circ \rho_i^{-1} \circ \pi)$ is satisfied in some F_j. Thus $(R, \rho_j \circ \sigma_i^j \circ \rho_i^{-1} \circ \pi)$ is satisfied in some G_j.

$\mathcal{D}^{\circledast}$ is a model (2)



2. [...] Thus $(R, \rho_j \circ \sigma_i^j \circ \rho_i^{-1} \circ \pi)$ is satisfied in some G_j.
3. **Magic formula:** $\rho_j \circ \sigma_i^j \circ \rho_i^{-1} = \tau_i^j$
4. Thus $(R, \tau_i^j \circ \pi)$ satisfied in G_j
5. Then $(R, \tau^* \circ \tau_i^j \circ \pi) = (R, \tau^* \circ \pi)$ satisfied in $\tau^*(G_j)$.
6. Since $\pi(body(R))$ was **stable** in G_i, we have $\tau^* \circ \pi = \pi$.
7. We conclude with (R, π) satisfied in $\tau^*(G_j)$, and thus in $\mathcal{D}^{\circledast}$. □

A finitely universal model is a (BCQ) representative

Theorem ((BCQ) representative)

A finitely universal model M of a KB $\mathcal{K}$ is a (BCQ) representative of $\mathcal{K}$.

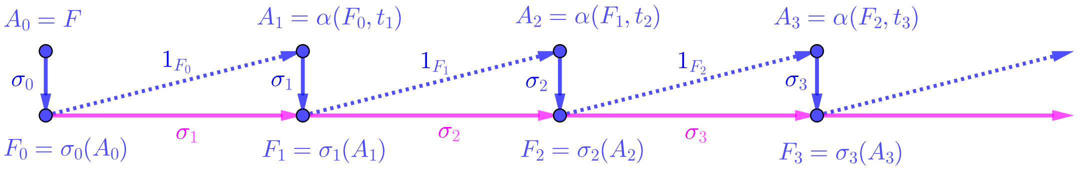
($\Rightarrow$) If $M \models Q$ then $\mathcal{K} \models Q$.

1. Let $\sigma : Q \to M$. Since Q **finite**, then $Q' = \sigma(Q)$ finite and thus (**finitely universal**) universal.
2. For any model M' of $\mathcal{K}$, since Q' is universal, we have $\sigma' : Q' \to M'$.
3. Thus $\sigma' \circ \sigma : Q \to M'$, which is a model of Q. □

($\Leftarrow$) If $\mathcal{K} \models Q$ then $M \models Q$.

1. For any fair derivation $\mathcal{D}$, we have $\sigma : Q \to \mathcal{D}^*$.
2. Since $\mathcal{D}^*$ is universal and M is a **model**, we have $\sigma' : \mathcal{D}^* \to M$.
3. Then $\sigma' \circ \sigma : Q \to M$. □

Preliminary: treewidth and monotonic derivations

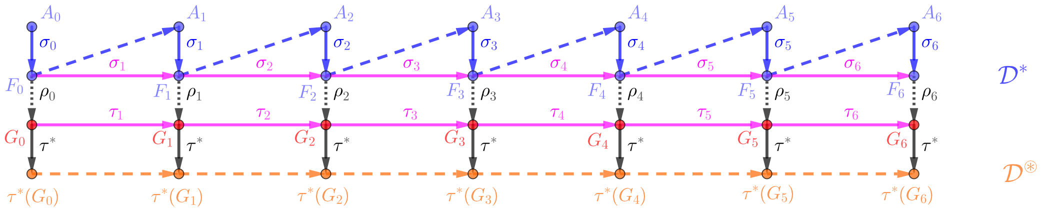


Theorem

If $\mathcal{D}$ is a monotonic derivation with bounded treewidth k, then $\mathcal{D}^*$ has treewidth $\leq k$.

1. Let us consider any finite subset A of $\mathcal{D}^*$. Since $\mathcal{D}$ is **monotonic**, there exists $i \in \mathfrak{I}$ such that $A \subseteq F_i$ (where is Wally).
2. Since $A \subseteq F_i$, we have $tw(A) \leq tw(F_i)$, and since $\mathcal{D}$ has bounded treewidth k, we have $tw(F_i) \leq k$. Thus $tw(A) \leq k$.
3. We conclude with the compactness theorem: $tw(\mathcal{D}^*) \leq k$ □

Treewidth and robust aggregation

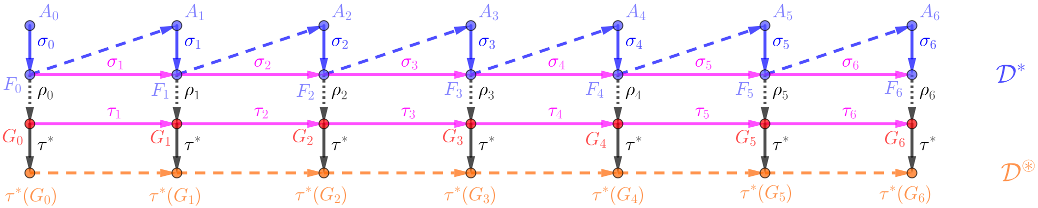


Theorem

If $\mathcal{D}$ is a derivation with bounded treewidth k, then $\mathcal{D}^{\circledast}$ has treewidth $\leq k$.

1. Let us consider any finite subset A of $\mathcal{D}^{\circledast}$. There is $i \in \mathfrak{I}$ st $A \subseteq G_i$ (Wally lemma).
2. Since $A \subseteq G_i$, we have $tw(A) \leq tw(G_i)$, since G_i is isomorphic to F_i we have $tw(G_i) = tw(F_i)$, and since $\mathcal{D}$ has btw k, we have $tw(F_i) \leq k$. Thus $tw(A) \leq k$.
3. We conclude with the compactness theorem: $tw(\mathcal{D}^*) \leq k$ □

Better yet: intermittently bounded treewidth (1)



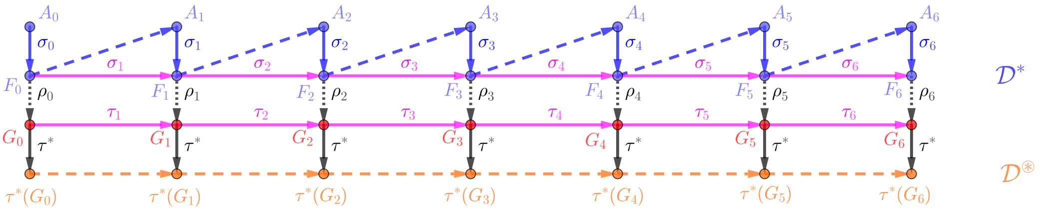
Definition

A derivation $\mathcal{D}$ has (**uniform**) bounded treewidth k when each F_i has treewidth $\leq k$. It has **intermittent** bounded treewith k when an infinite number of F_i have treewidth $\leq k$.

Theorem

If $\mathcal{D}$ is a derivation with intermittent bounded treewidth k, then $\mathcal{D}^{\circledast}$ has treewidth $\leq k$.

Better yet: intermittently bounded treewidth (2)



Theorem

If $\mathcal{D}$ is a derivation with intermittent bounded treewidth k, then $\mathcal{D}^{\circledast}$ has treewidth $\leq k$.

1. Let us consider any finite subset A of $\mathcal{D}^{\circledast}$. There exists $i \in \mathfrak{I}$ st, for any $j \geq i \in \mathfrak{I}$, $A \subseteq G_j$ (Wally lemma).
2. Since $\mathcal{D}$ has intermittent bounded treewidth k, there exists $q \geq i$ such that $tw(F_q) \leq k$.
3. We conclude as previously, working with G_q instead of G_i . . . □

Finishing touches (1)

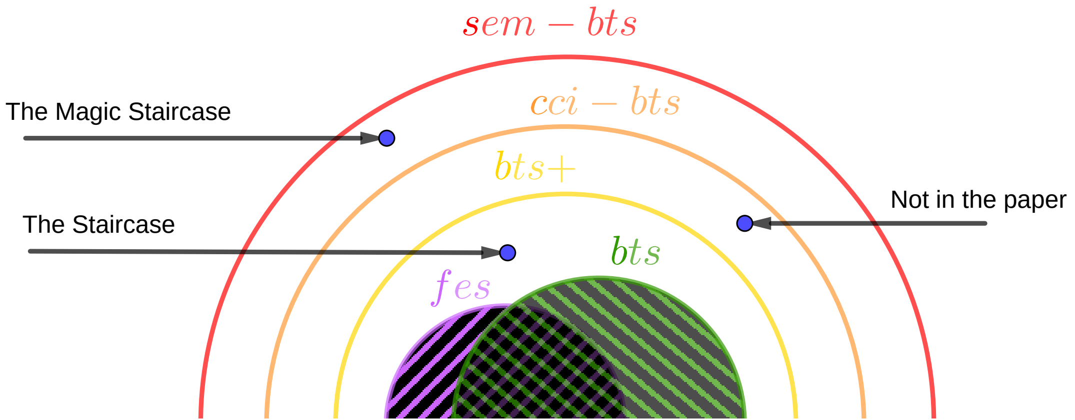
1. If $\mathcal{D}$ is a derivation with intermittent bounded treewidth k, then $\mathcal{D}^{\circledast}$ is a finitely universal model (and thus a BCQ representative) of treewidth $\leq k$.

Theorem (Basically (Cou90) + (BagLecMugSal11))

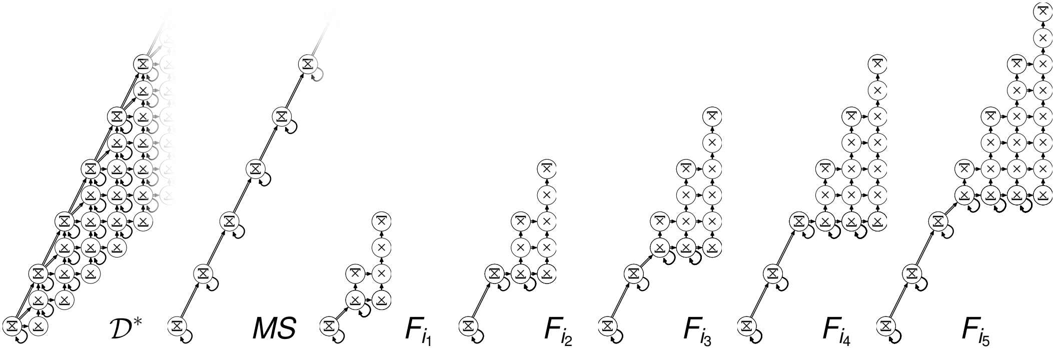
CQ entailment is decidable for knowledge bases admitting a finitely universal model of finite treewidth.

2. A ruleset $\mathcal{R}$ is said **fes** when, for every F, $(F, \mathcal{R})$ admits a finite universal model. It is said **bts** when, for every F, there is a **monotonic** derivation from $(F, \mathcal{R})$ (,*e.g. a restricted chase*) having uniformly bounded treewidth.
3. CQ entailment is decidable for KBs having fes or bts rulesets.
4. fes and bts are not comparable.

Finishing touches (3)



The Inflating Elevator



1. **Treewidth:** the core chase derivation $\mathcal{D}$ has unbounded treewidth, and $\mathcal{D}^*$ has infinite treewidth. No derivation with bounded treewidth can be obtained.
2. **Finite treewidth universal model:** The infinite atomset *MS* is a universal model of the inflating elevator KB, and it has treewidth 1.

Quasimodels

1. $\mathcal{D}^{\circledast}$ is a finitely universal model.
2. a finitely universal model is a (BCQ) representative
3. $\mathcal{D}^{*}$ is a universal (not a model), but a (BCQ) representative.

Objective

Find a nice characterization of a **quasimodel** *such that:*

- *$\mathcal{D}^{*}$ is a quasimodel*
- *a universal quasimodel is a (BCQ) representative*

1. is a finitely universal quasimodel a (BCQ) representative ?
2. is CQ entailment decidable when $\mathcal{K}$ admits a finitely universal quasimodel of finite treeewidth?
3. are all (BCQ) representatives finitely universal quasimodels?

Semantic BTS

1. A ruleset $\mathcal{R}$ is said **cci-bts** when, for every F, there is a derivation from $(F, \mathcal{R})$ (*,e.g. a core chase*) having intermittent bounded treewidth.
2. A ruleset $\mathcal{R}$ is said **sem-bts** when, for every F, there exists a finitely universal model of $(F, \mathcal{R})$ with finite treewidth.
3. the **magic staircase** rules are sem-bts, but not cci-bts.

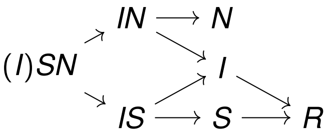
Remark

In the magic staircase core derivation, neither the F_i nor the G_i have (uniform or intermittent) bounded treewidth. However, the $\tau^*(G_i)$ have bounded treewidth.

1. see that if the $\tau^*(G_i)$ have intermittent bounded treewidth, then $\mathcal{D}^{\circledast}$ has finite treewidth; this leads to a new decidable class cci-bts $\subset$ **wf-cci-bts** $\subseteq$ sem-bts.
2. wouldn't it be nice to have wf-cci-bts = sem-bts?

Building infinite cores

1. the initial goal of well-founded aggregation was to generate a $\mathcal{D}^{\circledast}$ smaller than $\mathcal{D}^{*}$, hoping that in the case of a core chase, it would be a **core.**
2. **Infinite cores are tricky:** for finite atomsets, there is numerous definitions of cores that are all equivalent. For infinite atomsets, however, those different definitions lead to different notions of cores (**DBLP:journals/dm/Bauslaugh95**).



Failure

Neither $\mathcal{D}^{*}$ nor $\mathcal{D}^{\circledast}$ are ensured to be cores, whatever the definition.

1. generalize our general framework to take into account the stable chase (**DBLP:conf/icdt/CarralK0OR18**).