Exercise 1

SAT-Solving

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Exercise 1.1

Given an interpretation I and a formula F. Compute F^I:

	I	F
a)	$(1, 2, 3)$	$(1 \rightarrow -3) \leftrightarrow (2 \wedge -1 \wedge 3)$
b)	$(-1, 2, 3)$	
c)	$(-1, -2, -3, -4, -5)$	$\Big((-2 \vee 1) \rightarrow \big(3 \leftrightarrow (5 \vee 2)\big)\Big) \leftarrow (5 \wedge -3 \wedge 4)$
d)	$(-1, -2, -3, 4, 5)$	
e)	$(1, 2, -3, 4, 5)$	

Exercise 1.2

In the lecture we learnt that there exist different variants of SAT problems. This exercise will illustrate that. Let $F = 1 \wedge \Big(-1 \rightarrow \big(2 \vee 3 \leftrightarrow (-4 \wedge 2)\big)\Big)$. Solve F as a decision, search and all models variant.

Exercise 1.3

Find a model for F_1 and F_2.

$$
\begin{aligned}
F_1 = \; & -1 \\
\wedge \; & (2 \leftrightarrow 1) \\
\wedge \; & (2 \vee 3) \\
\wedge \; & (3 \rightarrow -2 \wedge -4) \\
\wedge \; & (4 \vee 5 \vee 6) \\
\wedge \; & (5 \rightarrow 7 \wedge 8) \\
\wedge \; & (-7 \vee 8) \\
\wedge \; & (5 \leftrightarrow -8)
\end{aligned}
\qquad
\begin{aligned}
F_2 = \; & (1 \vee -2) \\
\wedge \; & (1 \rightarrow 3) \\
\wedge \; & (-3 \vee 2) \\
\wedge \; & (-2 \vee (4 \wedge 5)) \\
\wedge \; & (4 \leftrightarrow -3) \\
\wedge \; & (5 \vee 6 \vee -2) \\
\wedge \; & (5 \leftrightarrow -6) \\
\wedge \; & (-1 \rightarrow (3 \vee 6))
\end{aligned}
$$

After finding a model on your own, try to find a model with the help of the basic algorithm presented in the lecture.

Exercise 1.4

1. Give the set of subformulas of F_1 and F_2.

$$
\begin{aligned}
F_1 &= \neg(p_1 \wedge (p_2 \rightarrow \neg p_3)) \\
F_2 &= (p_1 \leftrightarrow \neg p_2) \vee (p_2 \wedge p_1)
\end{aligned}
$$

2. Are the following statements correct? Proof your answer.

   (a) If all subformulas of F are satisfiable then F is also satisfiable.

   (b) A formula F is satisfiable if and only if at least one subformula is satisfiable.

Exercise 1.5

1. Use the semantic equivalences presented in the lecture (slide 17) to transform stepwise the following formulas. The resulting formulas have to contain at most the $\wedge$, $\vee$ and $\neg$ connectives (and as few as possible).
   (a) $F_1 = \neg\neg\neg(p_1 \vee p_2)$
   (b) $F_2 = (p_1 \wedge p_2) \rightarrow p_3$
   (c) $F_3 = (p_1 \leftrightarrow \neg p_2) \vee (p_2 \wedge p_1)$
2. Proof that $F \rightarrow G \equiv \neg F \vee G$ holds.