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Introduction to Formal Concept Analysis Exercise Sheet 9, Winter Semester 2017/18
Exercise 1 (attribute exploration)
This is a theme from elementary geometry: discussing <i>pairs of squares</i> . Here are two possible such pairs:
The two squares on the left overlap, because they have an inner point in common. They are also parallel in the sense that each side of one square is parallel to some side of the other. The two squares in the second diagram do not have these properties. Instead, they have a common vertex (but no common edge, not even a common segment of an edge). They are not disjoint, since they share a point.
We have collected a small list consisting of six attributes,
$overlap,\ parallel,\ disjoint,\ common\ vertex,\ common\ edge,\ common\ segment,$
and we have seen an object (a pair of squares) with the attribute combination
$overlap,\ parallel,$
and another one with the attribute "combination"
$common\ vertex.$
You may wonder which other attribute combinations are possible. Some, like
$overlap,\ parallel,\ disjoint,\ common\ vertex,$
are obviously not.
The idea of going through all possible cases is not very inviting. You would have to check $2^n$ cases for $n$ attributes, which makes already 64 for our toy example. What one usually does is to come up with more examples, like

hoping to have intuitively included all possibilities.

Use attribute exploration, starting with a formal context containing all the above examples as objects and perform attribute exploration to find out which implications hold among the six attributes and find an example set which refutes all other implications.