Experience Based Nonmonotonic Reasoning

Daniel Borchmann

TU Dresden

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We are waiting for the tram, and it's a sunny day \dots

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Observation

Non-monotonic behavior!

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How to model?

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No prior logical knowledge

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- Previous experiences

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Non-monotonic behavior!

How to model?

- No prior logical knowledge
- Previous experiences

Decide what to expect based on previous experiences in similar situations

Approach

Formalize this situation using formal concept analysis (FCA)

	sunny	sirens	tram on time
Day 1	×		×
Day 2			×
Day 3	×		×
Day 4	×	×	×
Day 5	×	×	
Day 6	×		×

	sunny	sirens	tram on time
Day 1	×		×
Day 1 Day 2			×
Day 3 Day 4	×		×
Day 4	×	×	×
Day 5 Day 6	×	×	
Day 6	×		×

Example

	sunny	sirens	tram on time
Day 1	×		×
Day 1 Day 2 Day 3 Day 4			×
Day 3	×		×
Day 4	×	×	×
Day 5 Day 6	×	×	
Day 6	×		×

Example

On what days was it sunny and the tram was on time?

	sunny	sirens	tram on time
Day 1	×		×
Day 1 Day 2 Day 3 Day 4			×
Day 3	×		×
Day 4	×	×	×
Day 5 Day 6	×	×	
Day 6	×		×

Example

On what days was it sunny and the tram was on time?

 $\{$ sunny, tram on time $\}'$

	sunny	sirens	tram on time
Day 1	×		×
Day 1 Day 2			×
Day 3 Day 4 Day 5 Day 6	×		×
Day 4	×	×	×
Day 5	×	×	
Day 6	×		×

Example

On what days was it sunny and the tram was on time?

 $\{\, \mathsf{sunny}, \mathsf{tram} \; \mathsf{on} \; \mathsf{time} \,\}' = \{\, \mathsf{Day} \; \mathsf{1}, \mathsf{Day} \; \mathsf{3}, \mathsf{Day} \; \mathsf{4}, \mathsf{Day} \; \mathsf{6} \,\}$

	sunny	sirens	tram on time
Day 1	×		×
Day 1 Day 2			×
Day 3 Day 4	×		×
Day 4	×	×	×
Day 5 Day 6	×	×	
Day 6	×		×

Example

	sunny	sirens	tram on time
Day 1	×		×
Day 1 Day 2 Day 3 Day 4			×
Day 3	×		×
Day 4	×	×	×
Day 5 Day 6	×	×	
Day 6	×		×

Example

What observations have Day 1 and Day 5 in common?

	sunny	sirens	tram on time
Day 1	×		×
Day 2			×
Day 3	×		×
Day 4	×	×	×
Day 5	×	×	
Day 6	×		×

Example

What observations have Day 1 and Day 5 in common?

$$\{ Day 1, Day 5 \}'$$

	sunny	sirens	tram on time
Day 1 Day 2 Day 3 Day 4 Day 5 Day 6	×		×
Day 2			×
Day 3	×		×
Day 4	×	×	×
Day 5	×	×	
Day 6	×		×

Example

What observations have Day 1 and Day 5 in common?

$$\{ \text{ Day 1, Day 5 } \}' = \{ \text{ sunny } \}$$

	sunny	sirens	tram on time
Day 1	×		×
Day 1 Day 2			×
Day 3	×		×
Day 4	×	×	×
Day 5 Day 6	×	×	
Day 6	×		×

	sunny	sirens	tram on time
Day 1	×		×
Day 2			×
Day 3	×		×
Day 4	×	×	×
Day 5 Day 6	×	×	
Day 6	×		×

Idea

If it is sunny, it *normally* follows that the tram is on time if this has happened *often enough* in similar situations.

	sunny	sirens	tram on time
Day 1	×		×
Day 1 Day 2			×
Day 3	×		×
Day 4	×	×	×
Day 5	×	×	
Day 6	×		×

Idea

If it is sunny, it *normally* follows that the tram is on time if this has happened *often enough* in similar situations.

True for Day 1, Day 3, Day 4, Day 6

	sunny	sirens	tram on time
Day 1	×		×
Day 2			×
Day 3	×		×
Day 4	×	×	×
Day 5	×	×	
Day 6	×		×

Idea

If it is sunny, it *normally* follows that the tram is on time if this has happened *often enough* in similar situations.

- True for Day 1, Day 3, Day 4, Day 6
- Not true for Day 5

	sunny	sirens	tram on time
Day 1	×		×
Day 1 Day 2 Day 3 Day 4			×
Day 3	×		×
Day 4	×	×	×
Day 5 Day 6	×	×	
Day 6	×		×

	sunny	sirens	tram on time
Day 1	×		×
Day 1 Day 2			×
Day 3	×		×
Day 4	×	×	×
Day 5	×	×	
Day 6	×		×

Observation

If is sunny and there are sirens, it is not clear whether the tram will be on time

	sunny	sirens	tram on time
Day 1	×		×
Day 1 Day 2			×
Day 3	×		×
Day 4	×	×	×
Day 5	×	×	
Day 6	×		×

Observation

If is sunny and there are sirens, it is not clear whether the tram will be on time

True on Day 4

	sunny	sirens	tram on time
Day 1	×		×
Day 1 Day 2			×
Day 3 Day 4	×		×
Day 4	×	×	×
Day 5 Day 6	×	×	
Day 6	×		×

Observation

If is sunny and there are sirens, it is not clear whether the tram will be on time

- ► True on Day 4
- Not true on Day 5

	sunny	sirens	tram on time
Day 1	×		×
Day 2			×
Day 3	×		×
Day 4	×	×	×
Day 5 Day 6	×	×	
Day 6	×		×

Observation

If is sunny and there are sirens, it is not clear whether the tram will be on time

- ► True on Day 4
- Not true on Day 5

Not enough evidence!

Formalize notion of "normally follows"

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Approach

Use notion of confidence:

Formalize notion of "normally follows"

Approach

Use notion of *confidence*: define for *X*, *Y* sets of *attributes*

$$\operatorname{conf}(X \to Y) := egin{cases} 1 & X' = \varnothing \\ \frac{|(X \cup Y)'|}{|X'|} & \operatorname{otherwise} \end{cases}$$

as the *confidence* of $X \rightarrow Y$.

Formalize notion of "normally follows"

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as the *confidence* of $X \to Y$.

Example

$$conf(\{ sunny \} \rightarrow \{ tram \ on \ time \}) = 4/5$$

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Approach

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as the *confidence* of $X \to Y$.

Example

$$\begin{split} &conf(\{\,sunny\,\} \to \{\,tram\;on\;time\,\}) = 4/5\\ &conf(\{\,sunny,\,sirens\,\} \to \{\,tram\;on\;time\,\}) = 1/2 \end{split}$$

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Use notion of *confidence*: define for *X*, *Y* sets of *attributes*

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as the *confidence* of $X \to Y$.

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Approach

Consider implications with high confidence as "normally true"

Approach does not seem to be new

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- Should exist similar approaches, much better investigated

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Still interesting!

Shows connection between FCA and NMR

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- FCA provides interfaces to Data Mining

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- Approach may be used to mine "non-monotonic rules" from data

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- Should exist similar approaches, much better investigated

- Shows connection between FCA and NMR
- FCA provides interfaces to Data Mining
- Approach may be used to mine "non-monotonic rules" from data
- Closer look on connections between this approach and existing ones

Thank You