



FOUNDATIONS OF DATABASES AND QUERY LANGUAGES

Lecture 12: Evaluation of Datalog (2)

Markus Krötzsch

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Overview

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2. First-order queries
3. Complexity of query answering
4. Complexity of FO query answering
5. Conjunctive queries
6. Tree-like conjunctive queries
7. Query optimisation
8. Conjunctive Query Optimisation / First-Order Expressiveness
9. First-Order Expressiveness / Introduction to Datalog
10. Expressive Power and Complexity of Datalog
11. Optimisation and Evaluation of Datalog
12. Evaluation of Datalog (2)
13. Path queries
14. Outlook: database theory in practice

See course homepage [[⇒ link](#)] for more information and materials

Review: Datalog Evaluation

A rule-based recursive query language

```
father(alice, bob)
mother(alice, carla)
  Parent(x, y) ← father(x, y)
  Parent(x, y) ← mother(x, y)
SameGeneration(x, x)
SameGeneration(x, y) ← Parent(x, v) ∧ Parent(y, w) ∧ SameGeneration(v, w)
```

Perfect static optimisation for Datalog is undecidable

Datalog queries can be evaluated bottom-up or top-down

Simplest practical bottom-up technique: semi-naive evaluation

Semi-Naive Evaluation: Example

$$\begin{array}{l} e(1, 2) \quad e(2, 3) \quad e(3, 4) \quad e(4, 5) \\ (R1) \quad T(x, y) \leftarrow e(x, y) \\ (R2.1) \quad T(x, z) \leftarrow \Delta_T^i(x, y) \wedge T^i(y, z) \\ (R2.2') \quad T(x, z) \leftarrow T^{i-1}(x, y) \wedge \Delta_T^i(y, z) \end{array}$$

How many body matches do we need to iterate over?

$$\begin{array}{ll} T_P^0 = \emptyset & \text{initialisation} \\ T_P^1 = \{T(1, 2), T(2, 3), T(3, 4), T(4, 5)\} & 4 \times (R1) \\ T_P^2 = T_P^1 \cup \{T(1, 3), T(2, 4), T(3, 5)\} & 3 \times (R2.1) \\ T_P^3 = T_P^2 \cup \{T(1, 4), T(2, 5), T(1, 5)\} & 3 \times (R2.1), 2 \times (R2.2') \\ T_P^4 = T_P^3 = T_P^\infty & 1 \times (R2.1), 1 \times (R2.2') \end{array}$$

In total, we considered 14 matches to derive 11 facts

Semi-Naive Evaluation: Full Definition

In general, a rule of the form

$$H(\vec{x}) \leftarrow e_1(\vec{y}_1) \wedge \dots \wedge e_n(\vec{y}_n) \wedge I_1(\vec{z}_1) \wedge I_2(\vec{z}_2) \wedge \dots \wedge I_m(\vec{z}_m)$$

is transformed into m rules

$$H(\vec{x}) \leftarrow e_1(\vec{y}_1) \wedge \dots \wedge e_n(\vec{y}_n) \wedge \Delta_{I_1}^i(\vec{z}_1) \wedge I_2^i(\vec{z}_2) \wedge \dots \wedge I_m^i(\vec{z}_m)$$

$$H(\vec{x}) \leftarrow e_1(\vec{y}_1) \wedge \dots \wedge e_n(\vec{y}_n) \wedge I_1^{i-1}(\vec{z}_1) \wedge \Delta_{I_2}^i(\vec{z}_2) \wedge \dots \wedge I_m^i(\vec{z}_m)$$

...

$$H(\vec{x}) \leftarrow e_1(\vec{y}_1) \wedge \dots \wedge e_n(\vec{y}_n) \wedge I_1^{i-1}(\vec{z}_1) \wedge I_2^{i-1}(\vec{z}_2) \wedge \dots \wedge \Delta_{I_m}^i(\vec{z}_m)$$

Advantages and disadvantages:

- Huge improvement over naive evaluation
- Some redundant computations remain (see example)
- Some overhead for implementation (store level of entailments)

Top-Down Evaluation

Idea: we may not need to compute all derivations to answer a particular query

Example:

$$\begin{array}{l} e(1, 2) \quad e(2, 3) \quad e(3, 4) \quad e(4, 5) \\ (R1) \quad T(x, y) \leftarrow e(x, y) \\ (R2) \quad T(x, z) \leftarrow T(x, y) \wedge T(y, z) \\ \text{Query}(z) \leftarrow T(2, z) \end{array}$$

The answers to Query are the T-successors of 2.

However, bottom-up computation would also produce facts like $T(1, 4)$, which are neither directly nor indirectly relevant for computing the query result.

Assumption

For all techniques presented in this lecture, we assume that the given Datalog program is safe.

- This is without loss of generality (as shown in exercise).
- One can avoid this by adding more cases to algorithms.

Query-Subquery (QSQ)

QSQ is a technique for organising top-down Datalog query evaluation

Main principles:

- Apply **backward chaining/resolution**: start with query, find rules that can derive query, evaluate body atoms of those rules (subqueries) recursively
- Evaluate intermediate results **“set-at-a-time”** (using relational algebra on tables)
- Evaluate queries in a **“data-driven”** way, where operations are applied only to newly computed intermediate results (similar to idea in semi-naive evaluation)
- **“Push”** variable bindings (constants) from heads (queries) into bodies (subqueries)
- **“Pass”** variable bindings (constants) **“sideways”** from one body atom to the next

Details can be realised in several ways.

Adornments

To guide evaluation, we distinguish **free** and **bound** parameters in a predicate.

Example: if we want to derive atom $T(2, z)$ from the rule $T(x, z) \leftarrow T(x, y) \wedge T(y, z)$, then x will be bound to 2, while z is free.

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We use **adornments** to note the free/bound parameters in predicates.

Example:

$$T^{bf}(x, z) \leftarrow T^{bf}(x, y) \wedge T^{bf}(y, z)$$

- since x is bound in the head, it is also bound in the first atom
- any match for the first atom binds y , so y is bound when evaluating the second atom (in left-to-right evaluation)

Adornments: Examples

The adornment of the head of a rule determines the adornments of the body atoms:

$$R^{bbb}(x, y, z) \leftarrow R^{bbf}(x, y, v) \wedge R^{bbb}(x, v, z)$$

$$R^{fbf}(x, y, z) \leftarrow R^{fbf}(x, y, v) \wedge R^{bbf}(x, v, z)$$

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The order of body predicates matters affects the adornment:

$$S^{fff}(x, y, z) \leftarrow T^{ff}(x, v) \wedge T^{ff}(y, w) \wedge R^{bbf}(v, w, z)$$

$$S^{fff}(x, y, z) \leftarrow R^{fff}(v, w, z) \wedge T^{fb}(x, v) \wedge T^{fb}(y, w)$$

↪ For optimisation, some orders might be better than others

Auxiliary Relations for QSQ

To control evaluation, we store intermediate results in auxiliary relations.

When we “call” a rule with a head where some variables are bound, we need to provide the bindings as input

↪ for adorned relation R^α , we use an auxiliary relation input_R^α

↪ arity of input_R^α = number of b in α

The result of calling a rule should be the “completed” input, with values for the unbound variables added

↪ for adorned relation R^α , we use an auxiliary relation output_R^α

↪ arity of output_R^α = arity of R (= length of α)

Auxiliary Relations for QSQ (2)

When evaluating body atoms from left to right, we use supplementary relations sup_i

\leadsto bindings required to evaluate rest of rule after the i th body atom

\leadsto the first set of bindings sup_0 comes from input input_R^α

\leadsto the last set of bindings sup_n go to output output_R^α

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Example:

$$\begin{array}{ccccccc} T^{bf}(x, z) & \leftarrow & T^{bf}(x, y) & \wedge & T^{bf}(y, z) & & \\ & & \uparrow & & \searrow \uparrow & & \searrow \\ \text{input}_T^{bf} & \Rightarrow & \text{sup}_0[x] & & \text{sup}_1[x, y] & & \text{sup}_2[x, z] \Rightarrow \text{output}_T^{bf} \end{array}$$

- $\text{sup}_0[x]$ is copied from $\text{input}_T^{bf}[x]$ (with some exceptions, see exercise)
- $\text{sup}_1[x, y]$ is obtained by joining tables $\text{sup}_0[x]$ and $\text{output}_T^{bf}[x, y]$
- $\text{sup}_2[x, z]$ is obtained by joining tables $\text{sup}_1[x, y]$ and $\text{output}_T^{bf}[y, z]$
- $\text{output}_T^{bf}[x, z]$ is copied from $\text{sup}_2[x, z]$

(we use "named" notation like $[x, y]$ to suggest what to join on; the relations are the same)

QSQ Evaluation

The set of all auxiliary relations is called a **QSQ template** (for the given set of adorned rules)

General evaluation:

- add new tuples to auxiliary relations until reaching a fixed point
- evaluation of a rule can proceed as sketched on previous slide
- in addition, whenever new tuples are added to a sup relation that feeds into an IDB atom, the input relation of this atom is extended to include all binding given by sup (may trigger subquery evaluation)

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Notation we will use:

- for an EDB atom A , we write A^I for table that consists of all matches for A in the database

Recursive QSQ

Recursive QSQ (QSQR) takes a “depth-first” approach to QSQ

Evaluation of single rule in QSQR:

Given: adorned rule r with head predicate R^α ; current values of all QSQ relations

- (1) Copy tuples input_R^α (that unify with rule head) to sup_0^r
- (2) For each body atom A_1, \dots, A_n , do:
 - If A_i is an EDB atom, compute sup_i as projection of $\text{sup}_{i-1}^r \bowtie A_i^T$
 - If A_i is an IDB atom with adorned predicate S^β :
 - (a) Add new bindings from sup_{i-1}^r , combined with constants in A_i , to input_S^β
 - (b) If input_S^β changed, recursively evaluate all rules with head predicate S^β
 - (c) Compute sup_i^r as projection of $\text{sup}_{i-1}^r \bowtie \text{output}_S^\beta$
- (3) Add tuples in sup_n^r to output_R^α

QSQR Algorithm

Given: a Datalog program P and a conjunctive query $q[\vec{x}]$ (possibly with constants)

(1) Create an adorned program P^a :

- Turn the query $q[\vec{x}]$ into an adorned rule

Query ^{$ff\dots f$} (\vec{x}) \leftarrow $q[\vec{x}]$

- Recursively create adorned rules from rules in P for all adorned predicates in P^a .

(2) Initialise all auxiliary relations to empty sets.

(3) Evaluate the rule Query ^{$ff\dots f$} (\vec{x}) \leftarrow $q[\vec{x}]$.

Repeat until no new tuples are added to any QSQR relation.

(4) Return output_{Query ^{$ff\dots f$}}

QSQR Transformation: Example

Predicates S (same generation), p (parent), h (human)

$$S(x, x) \leftarrow h(x)$$

$$S(x, y) \leftarrow p(x, w) \wedge S(v, w) \wedge p(y, v)$$

with query $S(1, x)$.

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Magic Sets

QSQ(R) is a **goal directed** procedure: it tries to derive results for a specific query.

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Can a bottom-up technique be goal-directed?

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Can a bottom-up technique be goal-directed?

→ yes, by magic

Magic Sets

- “Simulation” of QSQ by Datalog rules
- Can be evaluated bottom up, e.g., with semi-naive evaluation
- The “magic sets” are the sets of tuples stored in the auxiliary relations
- Several other variants of the method exist

Magic Sets as Simulation of QSQ

Idea: the information flow in QSQ(R) mainly uses join and projection

~> can we just implement this in Datalog?

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Could be expressed using rules:

$$\begin{aligned} \text{sup}_0(x) &\leftarrow \text{input}_T^{bf}(x) \\ \text{sup}_1(x, y) &\leftarrow \text{sup}_0(x) \wedge \text{output}_T^{bf}(x, y) \\ \text{sup}_2(x, z) &\leftarrow \text{sup}_1(x, y) \wedge \text{output}_T^{bf}(y, z) \\ \text{output}_T^{bf}(x, z) &\leftarrow \text{sup}_2(x, z) \end{aligned}$$

Magic Sets as Simulation of QSQ (2)

Observation: $\text{sup}_0(x)$ and $\text{sup}_2(x, z)$ are redundant. Simpler:

$$\begin{aligned}\text{sup}_1(x, y) &\leftarrow \text{input}_T^{bf}(x) \wedge \text{output}_T^{bf}(x, y) \\ \text{output}_T^{bf}(x, z) &\leftarrow \text{sup}_1(x, y) \wedge \text{output}_T^{bf}(y, z)\end{aligned}$$

We still need to “call” subqueries recursively:

$$\text{input}_T^{bf}(y) \leftarrow \text{sup}_1(x, y)$$

It is easy to see how to do this for arbitrary adorned rules.

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Constants in rule bodies must lead to bindings in the subquery.

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Example: the following rule is correctly adorned

$$R^{bf}(x, y) \leftarrow T^{bbf}(x, a, z)$$

This leads to the following rules using Magic Sets:

$$\begin{aligned} \text{output}_R^{bf}(x, y) &\leftarrow \text{input}_R^{bf}(x) \wedge \text{output}_T^{bfb}(x, a, y) \\ \text{input}_T^{bbf}(x, a) &\leftarrow \text{input}_R^{bf}(x) \end{aligned}$$

Note that we do not need to use auxiliary predicates sup_0 or sup_1 here, by the simplification on the previous slide.

Magic Sets: Summary

A goal-directed bottom-up technique:

- Rewritten program rules can be constructed on the fly
- Bottom-up evaluation can be semi-naive (avoid repeated rule applications)
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Nevertheless, a full materialisation might be better, if

- Database does not change very often (materialisation as one-time investment)
- Queries are very diverse and may use any IDB relation (bad for caching supplementary relations)

↪ semi-naive evaluation is still very common in practice

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 - **Production Rules** use “bottom-up rule reasoning with operational, non-monotonic built-ins”
 - **Recursive SQL Queries** are a syntactically restricted set of Datalog rules
- ↪ Different scenarios, different optimal solutions
- ↪ Not all implementations are complete (e.g., Prolog)

Datalog Implementation in Practice

Dedicated Datalog engines as of 2015:

- **DLV** Answer set programming engine with good performance on Datalog programs (commercial)
- **LogicBlox** Big data analytics platform that uses Datalog rules (commercial)
- **Datomic** Distributed, versioned database using Datalog as main query language (commercial)

Several RDF (graph data model) DBMS also support Datalog-like rules, usually with limited IDB arity, e.g.:

- **OWLIM** Disk-backed RDF database with materialisation at load time (commercial)
- **RDFox** Fast in-memory RDF database with runtime materialisation and updates (academic)

~> Extremely diverse tools for very different requirements

Summary and Outlook

Several implementation techniques for Datalog

- bottom up (from the data) or top down (from the query)
- goal-directed (for a query) or not

Top-down: Query-Subquery (QSQ) approach (goal-directed)

Bottom-up:

- naive evaluation (not goal-directed)
- semi-naive evaluation (not goal-directed)
- Magic Sets (goal-directed)

Next topics:

- Graph databases and path queries
- Applications